# A Simple Equation for Estimating the Expectation of Life at Old Ages\*

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## INTRODUCTION

There is much direct and indirect evidence that ages of older persons tend to be misreported, both when reported in censuses and records of deaths. Their ages are more likely to be exaggerated rather than understated, resulting in too high an estimate of the proportions living and/or deaths at old ages. This overstatement transfers some persons of relatively younger ages in which mortality is lower to older age groups with higher mortality. This, in general, leads to underestimation of mortality rates at old ages and, in turn, overestimation of the expectation of life, unless ages at death are exaggerated considerably more than ages of living persons, and thus tend to inflate mortality at very old ages.

Indeed, official expectations of life at old ages in some populations are very large relative to the mortality at young and middle ages. This is illustrated in Figure 1, in which the expectation of life at age 65, is plotted against the survival rate between ages 10 and  $65 \left(_{55}p_{10}\right)$ , for men and women in 40 populations drawn from the *Demographic Yearbook* 1974.<sup>2</sup> Populations in which e(65) is large relatively to  $_{55}p_{10}$  include El Salvador 1960–1, Mexico 1970, Puerto Rico 1969–71, and West Malaysia 1971. (These are denoted in Figure 1 by the numbers 1, 2, 3 and 4, respectively.) The relative excess of e(65) over  $_{55}p_{10}$  in those populations may be the result of age misreporting, although it is possible that the excess is partly due to under-registration of deaths at old ages.

One way to avoid the impact of age exaggeration on mortality at old ages is to choose a model life table that fits the mortality of young and middle-aged adults for which reported ages are more reliable than those of older persons, and to assume that mortality patterns at old ages follow the selected model life table. Although this may be the only possible procedure in some populations, this approach has some limitations. Association between mortality at old ages and that at relatively younger ages may not be very strong. In some populations mortality may deviate significantly from any standard model life tables.<sup>3</sup>

Therefore, there seems to be need for a method of estimating the expectation of life

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- <sup>1</sup> See, for example, R. B. Mazess, 'Health and longevity in Vilcabamba, Ecuador', *Journal of the American Medical Association* **240** (1978), p. 1781; Z. A. Medvedev, 'Caucasus and Altay longevity: a biological or social problem?' *Gerontologist* **14**, 1974, p. 381; M. A. Gibril, 'Some reporting errors in the 1973 Gambian Census'. M.Sc. thesis, University of London: Ira Rosenwaike, 'A new evaluation of United States census data on the extremely aged', *Demography* **16** (2), 1979, pp. 279–288.
- <sup>2</sup> United Nations, Department of Economic and Social Affairs, Statistical Office, *Demographic Yearbook*, 1048-
- <sup>3</sup> See for example, Noreen Goldman, 'Far Eastern patterns of mortality', *Population Studies* **34** (1), 1980, pp. 5–19.

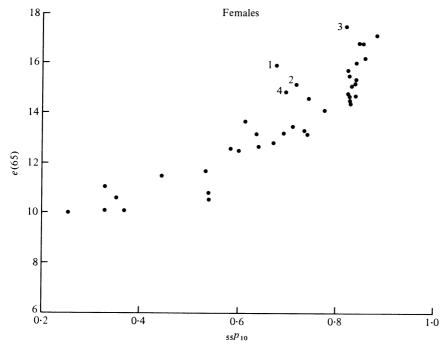


Fig. 1. Life expectancy at age 65 compared with survival rate between ages 10 and 65 for 40 selected female populations.

at old ages directly from data on mortality in old age, that is robust in the presence of substantial misreporting of ages. In this paper we will present a simple formula that can be used for the purpose.

## ASSUMPTIONS

The method is based on the following three assumptions: (1) there exists an age a such that most age transfer occurs *above*, *but not across* the age; (2) the mortality above age a is described by a Gompertz curve; and (3) the age distribution of the population above age a is approximately stable.

For the first assumption on how age is misreported we would expect substantial and systematic overstatement to be confined to the older age range. Since both the ages of older persons and of older decedents tend to be exaggerated, it may be possible to choose an age a for a population such that most age transfer of substantial magnitude occurs above that age and that transfer across the age is not significant or tends to cancel. In general, if the quality of age statistics is low, a relatively young age such as 55, 60 or 65 needs to be chosen, although in statistics of good quality reported ages may be reliable up to very high values such as 85, 90 or 95.

If the age a can be chosen properly, population measures for the whole open interval 'age a and above' will not be significantly biased by age misreporting, even though those for single years and five-year age groups above a are inaccurate. Then, the aggregate mortality rate for age a and above, denoted by M(a+), can be used as a relatively reliable measure of mortality in old age.

However, in populations with same mortality schedule M(a+) may be different if their age structures above age a are different. Therefore, some additional information on age

structure is needed. The growth rate for the open age interval, r(a+), is very useful for this purpose, because the age structure above age a is uniquely determined by the growth rate and the mortality schedule if the population aged a and over is stable, which is an assumption introduced later. Thus, based on the first assumption, M(a+) and r(a+) will be used for estimating e(a).

The second assumption is concerned with age patterns of mortality. It is assumed that the mortality at ages a above is described by the Gompertz function:

$$\mu(x) = \mu(a) e^{k(x-a)}, \quad \text{for every } x > a, \tag{1}$$

where  $\mu(x)$  is the force of mortality, or the mortality rate at exact age x, and k is a constant.<sup>4</sup> It is widely known that, for a number of populations with accurate age statistics,  $\log \mu(x)$  is well approximated by a linear function of age at old ages, as implied by (1).<sup>5</sup> Note also that, if mortality can be described by the Gompertz function, the survival rate from a to a+y is given by

$$_{y}P_{a}=\exp\Big(-\frac{\mu(a)}{k}\left(e^{ky}-1\right)\Big).$$

In order to test the validity of this assumption with respect to the estimation of the expectation of life, we have selected nine populations listed in Table 1 for which single-year data on age at death and ages of persons up to 95 or more are believed to be quite accurate, and we have compared the expectation of life directly calculated from the

Table 1. Expected length of life in the age intervals from 55, 70 and 85 to 95, for
selected countries with accurate single-year data on ages at death and ages of persons

		$_{40}{ m L}_{55}/{ m l}_{55}$		$_{25}\mathrm{L_{70}/l_{70}}$		$_{10}{ m L_{85}/l_{85}}$	
Country	Sex	Estimated*	Observed†	Estimated*	Observed†	Estimated*	Observed†
Australia, 1970-2	M	18.94	18.86	9.45	9.45	3.94	3.94
	F	23.72	23.69	12.30	12.26	4.66	4.66
Austria, 1960-2	M	18.70	18.69	9.38	9.39	3.64	3.64
	F	22.87	22.79	11.28	11.26	4.08	4.08
France, 1968	M	19.32	19.34	9.95	9.95	3.88	3.89
	F	24.48	24.48	12.66	12.61	4.58	4.59
Germany, 1910-11	M	15.56	16.58	8.19	8.20	3.29	3.29
	F	18.04	17.97	8.65	8.67	3.50	3.50
W. Germany, 1968	M	18.79	18.69	9.16	9.16	3.60	3.59
	F	22.95	22.83	11.24	11.21	4.01	4.01
E. Germany, 1976	M	19.47	19.36	9.24	9.24	3.54	3.54
	F	23.04	22.96	11.24	11.21	3.88	3.88
Japan, 1970	M	20.07	19.88	9.62	9.65	3.81	3.83
	F	23.68	23.53	11.84	11.79	4.30	4.31
Sweden, 1973-7	M	21.62	21.54	10.86	10.85	4.24	4.24
	F	25.78	25.75	13.49	13.43	4.90	4.90
Taiwan, 1931-5	M	14.07	14.05	7.34	7.30	3.36	3.32
	F	17.32	17.35	8.69	8.74	3.52	3.53

<sup>\*</sup> Estimated on the assumption of a Gompertz mortality pattern.

<sup>†</sup> Directly calculated from the observed age-specific death rates for single years of age.

<sup>&</sup>lt;sup>4</sup> For some theoretical bases of the Gompertz mortality pattern, see B. Strehler and A. Mildvan, 'General theory of mortality and aging', *Science* 132 (1960), pp. 14–21; D. Brillinger, 'A justification of some common law of mortality', *Transactions of the Society of Actuaries*, 13 (1961), pp. 116–119.

<sup>&</sup>lt;sup>5</sup> See, for example, A. J. Coale and Shiro Horiuchi, 'Age patterns of mortality for older persons', Office of Population Research, Princeton University (in preparation).

mortality rates for single-year age-groups with that obtained from the Gompertz curve fitted to the mortality rates. The function was fitted by ordinary least-squares regression of the logarithm of observed mortality rates on age.

However, since statistics by single year of age are not widely available nor always trustworthy for extremely high ages, e.g. those exceeding 100, the expectation of life is rarely computed without making an assumption about mortality at the end of the human life span. Therefore we have calculated the expected length of life in the interval between age a and 95, defined by

$$\int_a^{95} l(x) \, dx/l(a),$$

where l(x) is the survival rate up to age x, as a proxy of e(a), which is by definition

$$\int_a^\infty l(x)\,dx/l(a).$$

As is shown in Table 1, the expected length of life from age 55, 70 and 85 up to 95 is very accurately estimated from the Gompertz curve fitted to the observed death rates. This suggests that the Gompertz curve may validly be used for estimating the expectation of life at old ages.

The third assumption is concerned with age distribution. Even in a population in which the entire age structure is far from stable, the age distribution of the elderly may be well approximated by a stable distribution. In general, the assumption of stability for the older population seems more valid for developing than for developed countries, because the cohorts that are currently very old in many developing countries were born before the rapid decline in fertility and child mortality of recent decades.

Figure 2 suggests that the assumption may not be significantly violated even in developed countries in which mortality has been declining for a relatively long period. In the figure it is shown that the age distribution of the enumerated population and that

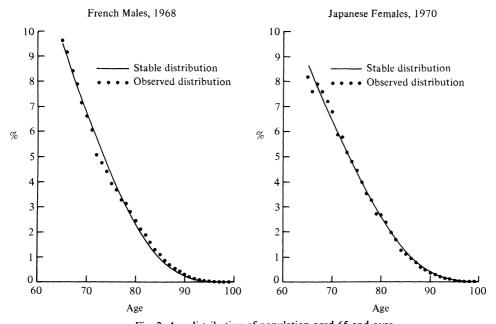


Fig. 2. Age distribution of population aged 65 and over.

of the corresponding stable population are quite close for the French males 1968 and Japanese females 1970.6

In combination with the Gompertz mortality pattern, the assumption of stability implies that the age structure at ages above a is given by

$$\frac{N(a+y)}{N(a)} = \exp\left(-ry - \frac{\mu(a)}{k}\left(e^{ky} - 1\right)\right),\,$$

where N(a) is the number of persons at age a and r is the growth rate of stable population.

## DERIVATION

The number of persons aged a and above in a stable population is

$$N(a+) = N(a) \int_0^\infty {_yP_a e^{-ry} dy},$$

where y is age measured with a as the origin. By expanding  $e^{-ry}$  in a Maclaurin series, we can approximate N(a+) by

$$N(a+) \doteq N(a) e(a) \left[ 1 - r\bar{y} + \frac{r^2}{2} (\bar{y}^2 + \sigma_y^2) \right],$$
 (2)

where  $\bar{y}$  and  $\sigma_y^2$  are respectively the mean and variance of y in the life table associated with the stable population.

Next, the number of deaths at ages a and over in a stable population is

$$D(a+) \doteq N(a) \int_0^\infty d_a(x) e^{-rx} dx,$$

where x is the difference between the age at death and a, and  $d_a(x)$  is defined by  $d_a(x) = {}_x P_a \mu(a+x)$ . Again, using a Maclaurin expansion, we find

$$D(a+) \doteq N(a) \left[ 1 - re(a) + \frac{r^2}{2} (e(a)^2 + \sigma_a^2) \right], \tag{3}$$

where  $\sigma_d^2$  is the variance of x in the life-table.

Dividing (3) by (2) and deleting terms in the third and higher powers of r, we can approximate the mortality rate at age a and above by

$$M(a+) \doteq \frac{1}{e(a)} \left[ 1 + r(\bar{y} - e(a)) + \frac{r^2}{2} (\bar{y} - e(a))^2 + \frac{r^2}{2} (\sigma_d^2 - \sigma_y^2) \right]$$
$$\doteq \frac{1}{e(a)} \left[ \exp(r(\bar{y} - e(a)) + \frac{r^2}{2} (\sigma_d^2 - \sigma_y^2)) \right].$$

At old ages the second term in the large bracket,  $(r^2/2)(\sigma_d^2 - \sigma_y^2)$ , is very small relative to the exponential term. Thus, deleting the second term and re-arranging the rest, we have

$$1/e(a) \doteq M(a+) \exp(r(e(a)-\bar{v})). \tag{4}$$

We see that if r = 0 in (4), the well-known stationary relation e(a) = 1/M(a+) is obtained. Note also that (4) is supposed to hold in any stable populations with reasonable life tables no matter whether or not the mortality schedules follow a Gompertz curve.

<sup>6</sup> The stable age distribution has been constructed from the Gompertz curve fitted to the observed mortality rates between ages 70 and 95, and the growth rate above age 65 for the intercensal period preceding the study year.

Now, we focus on  $e(a) - \overline{y}$  in (4). Let  $\delta = e(a) - \overline{y}$ . We know that, as e(a) approaches to zero, so does  $\delta$ . A functional relationship between  $\delta$  and e(a) that satisfies the condition is  $\delta = \beta [e(a)]^{\alpha}. \tag{5}$ 

In order to test the validity of (5) in Gompertz mortality schedules, we have constructed 32 Gompertzian life-tables for ages 50 and over by choosing levels 9, 13, 17 and 21 (corresponding to e(0) = 40, 50, 60 and 70, respectively, for females), for males and females, and for each one of four families of Coale and Demeny's model life tables, and have fitted a Gompertz function to the survival rates up to ages 50, 65 and 80, that is, l(50), l(65) and l(80), in those life tables. The Gompertz function parameters, k and l(50), have been computed from l(50), l(65) and l(80) by

$$\hat{k} = \frac{1}{15} \log \frac{(\log (l(80)/l(65)))}{(\log (l(65)/l(50)))}$$

and

$$\hat{\mu}(50) = \frac{\hat{k} \log (l(65)/l(50))}{1 - e^{15\hat{k}}} \ .$$

Figure 3 shows that  $\log \delta$  is almost linearly related to  $\log e(a)$  for ages 60, 70 and 80 in those Gompertzian life-tables, as implied by (5).

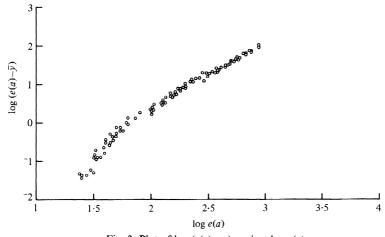


Fig. 3. Plot of  $\log (e(a) - \bar{y})$  against  $\log e(a)$ .

In practice we do not estimate  $\delta$  from e(a), which is the final outcome of the proposed method. A more practical alternative to (5) is obtained by replacing e(a), equivalently the inverse of M(a+) in a stationary population, with the inverse of M(a+) in a stable population. Thus we use  $\hat{\delta} = \beta [M(a+)]^{-\alpha}. \tag{6}$ 

Substituting (6) into (4), we obtain

$$1/\hat{e}(a) = M(a+) \exp(\beta r(a+)[M(a+)]^{-\alpha}). \tag{7}$$

This equation is proposed for estimating e(a) from M(a+) and r(a+). Hereafter, the expectation of life estimated by (7) will be called the DROI (Death Rate for the Open Interval) estimate.

In order to obtain appropriate values of  $\alpha$  and  $\beta$  in (7) we have generated 128 stable

<sup>7</sup> A. J. Coale and P. Demeny, Regional Model Life Tables and Stable Populations (Princeton: Princeton University Press, 1966).

populations aged 50 and over by combining 32 Gompertzian life-tables described above with four different positive annual growth rates, 1, 2, 3 and 4 per cent, respectively, and have performed regression analyses with the artificial data. In those analyses, log 1/(e(a) M(a+)) was regressed on  $r(a+)[M(a+)]^{-\alpha}$ , with the condition that the regression line pass through the origin, given various values of  $\alpha$ . The values of  $\alpha$  and  $\beta$  selected on the basis of regression results are presented in Table 2. As shown in the table, Equation (7) provides quite accurate estimates of e(a), with very small proportional errors. Although we recommend use of different combinations of  $\alpha$  and  $\beta$  for different ages under 65,  $\alpha = 1.4$  and  $\beta = 0.0951$  seem to provide very accurate estimates of the expectation of life at any age above and including 65.

			Percentile errors									
	Par	ameters			Percentage of absolute errors that are smaller							
Age	α	β	Maximum	Minimum	than 2 per cent	(N)	$R^2$					
50	1.0	0.2827	4.66	-3.37	57.0	(128)	0.96128					
55	1.1	0.2068	3.41	-2.71	83.6	(128)	0.96765					
60	1.2	0.1565	2.40	-2.03	97.7	(128)	0.97375					
65, 75, 85	1.4	0.0951	1.32	-2.36	99.5	(512)	0.98712					

Table 2. Parameter values used in Equation (7)

## **APPLICATIONS**

We have applied Equation (7) to statistics from some selected populations that may be divided into two groups: populations in which data on age are considered to be quite accurate (Group 1), and populations with seemingly unreasonably high life expectancies at old ages (Group 2). Populations in Group 2 have been chosen on the basis of Figure 1. For populations in Group 1, the DROI estimate of the expectation of life is supposed to agree with the officially reported figure quite well. For populations in Group 2, the DROI estimate is expected to be lower than the reported figure. Results for age 65 are presented in Table 3.

All the statistics were obtained from the *Demographic Yearbooks*. Under the heading 'official report', e(65)'s in the *Demographic Yearbook 1974* are shown. M(65+) was obtained by dividing the number of deaths at ages above 65 by the number of persons of those ages, and r(65+) was calculated for the intercensal period preceding the estimation year.

As is shown in Table 3, the DROI estimates of e(65) are close to those reported in the *Demographic Yearbook* for the populations in Group 1, but significantly lower than the official figures for those in Group 2. Those results seem to suggest that Equation (7) estimates the expectation of life at old ages accurately and is useful in correcting e(a) for age exaggeration among very old persons.

<sup>8</sup> United Nations, op. cit. in fn. 2.

<sup>&</sup>lt;sup>9</sup> M(65+) needs to be adjusted for completeness of death registration relative to that of census enumeration, if the relative completeness is believed to be significantly different from 100%. We have assumed that the registration of deaths is nearly complete in populations in Group 1. As for populations in Group 2, quite reliable and internally consistent estimates of completeness were obtained only for El Salvador, 1961, so that no adjustment has been made for the others. For El Salvadorian males and females, the completeness of death registration has been estimated to be 78% and 79%, respectively, using the stable method described in National Academy of Sciences, Committee on Population and Demography, *Demographic Estimation: A Manual on Indirect Techniques* (forthcoming), chapter 6, section B.

Table 3. Comparison of e(65) between official reports and DROI estimates for selected countries

		Official report New es		stimate		
		(1)	(2)	(3)	(4)	Difference
Country	Sex	e(65)	Year	e(65)	Year	(%)
			Group 1			
Belgium	M F	12.10 15.29	1968–72	12.26 15.88	1970	1.3 3.9
Canada	M F	13.63 16.71	1966	13.63 16.81	1966	0.0 0.6
France	M F	12.70 16.50	1968	12.96 17.25	1968	2.0 4.5
Italy	M F	13.30 16.15	1970–72	13.13 16.57	1971	-1.3 $2.6$
Japan	M F	11.86 14.59	1965	12.00 14.75	1965	1.2 1.1
Netherlands	M F	13.60 16.60	1971	13.30 16.30	1971	$-2.2 \\ -1.8$
Norway	M F	13.86 16.55	1966–70	13.86 17.14	1970	0.0 3.6
Switzerland	M F	13.31 16.30	1969–72	13.72 16.88	1970	3.1 3.6
U.K.	M F	12.10 16.00	1970–72	12.49 16.65	1971	3.2 4.1
			Group 2			
El Salvador	M F	14.27 15.86	1960–61	12.07 13.32	1961	-15.4 $-16.0$
Mexico	M F	14.17 15.22	1970	13.08 13.57	1970	-7.7 $-10.8$
Puerto Rico	M F	15.53 17.53	1969–71	14.21 15.48	1970	-8.5 -11.7
W. Malaysia	M F	11.84 14.85	1971	11.15 13.71	1970	- 5.8 - 7.7

Source: United Nations, Demographic Yearbook.

### DISCUSSION

There are several limitations and pitfalls in the proposed method. First, strictly speaking, statistics in many developing nations may not meet the basic requirement of the method that M(a+) and r(a+) are not distorted by age misreporting for the chosen age a. In most populations with defective age data there may well be age transfer across any age, so that M(a+) and r(a+) are inaccurate whatever value of a is chosen. However, the mortality and growth rates for the aggregated open interval 'a and above' are likely to be more reliable than those for one-year or five-year age-groups when there is a tendency for ages of older persons to be exaggerated. Therefore, the method seems to be useful at least in reducing the effects of age misreporting on the expectation of life at old ages and improving the accuracy of its estimate.

Secondly, it may not be easy to choose an appropriate age a. Although selecting a lower age might reduce the accuracy of the DROI estimate as shown in Table 2 and also jeopardize the assumptions of stable age distribution and Gompertz mortality pattern

in the open interval, raising the lower bound of the age interval increases the risk of age transfer across that bound, and perhaps biases the DROI estimate upward.

We suggest that the DROI estimates of the expectation of life at different ages be compared with each other, in order to locate a proper lower bound of the open interval. Such a detection of age a for Mexican females and Puerto Rican females, 1970, is illustrated in Table 4. In the table the DROI estimate of e(a) is shown for each age between 50 and 75 in five-year intervals and the value of e(65) that is at the same mortality level as the DROI estimate of e(a). The series of estimates of e(65) was computed from e(a)'s using Coale and Demeny's West Model life tables.<sup>10</sup>

	Mexican females		Puerto	Rican females	
Age	e(a)	e(65) estimated from $e(a)$	e(a)	e(65) estimated from $e(a)$	
50	25.27	13.77	25.55	13.93	
55	20.71	13.44	22.36	14.57	
60	17.24	13.69	19.15	15.25	
65	13.57	13.57	15.48	15.48	
70	11.22	14.57	12.37	16.03	
75	8.93	15.50	9.92	17.15	

Table 4. DROI estimates of expectation of life for Mexican females and Puerto Rican females, 1970

As is seen in the sequence of values of e(65) in Table 4, e(65), corresponding to the estimated e(a) for Mexican females remains substantially the same between ages 50 and 65, although it increases sharply with age after 65. The pattern seems to suggest that substantial effects of age exaggeration on the DROI estimate start to appear at ages above 65, so that 65 should be chosen as the lower bound of the open interval.

A very different pattern is seen for Puerto Rican females. As is shown in Table 4, e(65) corresponding to the DROI estimate of e(a) increases very steeply with age from 50 to 75. This suggests that a substantial amount of age exaggeration may begin around age 50 or even earlier. Perhaps the DROI estimate of e(65), 15.48, is still too high, even though it is significantly lower than the officially reported figure of 17.53.

Thirdly, the age distribution of some elderly populations may not be approximately stable. Improvement of mortality at old ages tends to make the actual age distribution steeper than the corresponding stable age distribution, thereby biasing the DROI estimate upward. This might be the reason why the DROI estimate of e(65) tends to be slightly higher than the corresponding official figure for the countries in Group 1 in Table 3. However, this type of bias is likely to be small, because, as was previously shown in Figure 2, the age structure of the elderly population does not differ from the stable distribution very much; even in such countries as France and Japan in which mortality has been declining for a long period.

Finally, any difference between the completeness of death registration and that of census enumeration will introduce biases into M(a+), and differential enumeration in two successive censuses may lead to over-or underestimation of r(a+). It should be noted, however, that several methods have been developed for estimating those two types of

<sup>&</sup>lt;sup>10</sup> A. J. Coale and P. Demeny, *op. cit.* in fn. 7. A revised version of Coale and Demeny's model life tables is presently in preparation, and we recommend that the new edition be used in analyses of mortality in old age. In the forthcoming version the segments of the present set of Coale and Demeny's life tables relating to old age will be modified on the basis of the Gompertz mortality pattern.

differential completeness simultaneously, and the techniques may be used to overcome those problems.<sup>11</sup>

In spite of those limitations, our analysis of the artificial data on stable populations and the real data on some selected populations seems to suggest that, if the method is used judiciously and possible pitfalls are borne in mind, it is useful in estimating the expectation of life at old ages from inaccurate age statistics. Equation (7) is also useful in closing life tables because the formula provides an estimate of e(a) for the open interval a+. One of the conventional solutions to the problem of constructing life tables for the highest age group is to equate e(a) to the reciprocal of M(a+), assuming the population above age a to be approximately stationary. Equation (7) allows us to use r(a+), an additional item of information often available for the highest age group, as well as M(a+) for improving the accuracy of the estimated value of e(a).

<sup>11</sup> S. H. Preston and K. Hill, 'Estimating the completeness of death registration', *Population Studies* 34 (2), 1980, pp. 349–366; S. H. Preston, A. J. Coale, T. J. Trussell and M. Weinstein, 'Estimating the completeness of reporting of adult deaths in populations that are approximately stable', *Population Index* 46 (2), 1980, pp. 179–202; N. Bennett and S. Horiuchi, 'Estimating the completeness of death registration in a closed population', *Population Index* 47 (2), 1981 (pp. 207–221); W. Brass, 'A procedure for comparing mortality measures calculated from intercensal survival with the corresponding estimates from registered deaths', *Asian and Pacific Census Forum*, 6 (2), 1979, pp. 5–7; T. J. Trussell and Jane Menken, 'Estimating the completeness of deaths and relative underenumeration in two successive censuses', *Asian and Pacific Census Forum* 6 (2), 1979, pp. 9–11.