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# Age-Specific Growth Rates: The Legacy of Past Population Dynamics

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Recent developments in population mathematics have focused attention on a function that is widely available but rarely examined: the set of age-specific growth rates in a population. In particular, this set of rates is sufficient for translating the current birth rate and age-specific mortality rates into the current age distribution. This growth-rate function contains all of the pertinent features of a population's demographic history that are required to relate major demographic functions for a particular period to one another. This article presents an expression for the age-specific growth rate and uses it to derive an equation for age distribution. We show how the value of the age-specific growth rate is determined by a population's demographic past and present various sets of growth rates corresponding to stylized demographic scenarios. Several noteworthy sets of growth rates observed in human populations are discussed. Finally, we explain why age-specific growth rates make it possible to determine the age distribution solely from information on current demographic conditions.

The current age structure of a population is a product of that population's past rates of fertility, mortality, and migration. The recent past is particularly important in determining the present age distribution, whereas traces of the rates that prevailed in the remote past become vanishingly small (Coale, 1957, 1972; Coale and Trussell, 1981; Kim, 1985, 1987; Kim and Sykes, 1976, 1978; Lopez, 1961, 1967; McFarland, 1969). In a stable population, the age structure can be expressed as a function of its *current* demographic rates because, by assumption, these rates also applied in the past. This accounts for much of the appeal of making stable population assumptions in demographic analysis, since in a stable population, there are explicit mathematical relations among current levels of all major demographic variables.

Recent developments in population mathematics have provided new and more generally applicable means for relating the current age structure to current regimes of demographic rates. These relations follow on the work of McKendrick (1926; McKendrick and Pai, 1910), von Foerster (1959), and Hoppensteadt (1975), among others. They proposed and elaborated a partial differential equation that relates the rates of change in population size with respect to age and time to the force of mortality at a moment in time. This differential equation is often called the von Foerster equation, although it was proposed first by McKendrick.

By integrating the partial differential equation with respect to age, Bennett and Horiuchi (1981) and Preston and Coale (1982) showed that the proportional age structure in *any* population can be expressed as a function of current demographic rates, namely, the birth rate, age-specific mortality rates, age-specific rates of net migration, and age-specific growth rates. The functional relationships hold regardless of the past history of the population. All of the pertinent features of that history that are needed to map current vital rates into current age distributions are contained in the set of age-specific growth rates, to which little importance has been attached in previous literature. Although age-specific growth rates do not directly represent any particular type of demographic behavior, such as fertility, mortality, or migration, they are widely available population measures (e.g., from successive censuses). The usefulness of the age-specific growth rate in demographic data analysis was illustrated in a variety of recent methodological developments (Bennett and Horiuchi, 1981, 1984; Coale, 1984; Coale, John, and Richards, 1985; Horiuchi and Coale, 1982; Preston, 1983; Preston and Bennett, 1983; Preston and Coale, 1982).

In this article, we focus attention on the age-specific growth-rate function itself. A readily interpretable expression for this function is provided. It makes explicit how age-specific growth rates reflect a population's demographic history. We show how this expression provides an alternative derivation of the new generalization of stable population relations. We then describe typical patterns of age-specific growth rates that correspond to particular regimes of demographic change and examine specific instructive sets of observed age-specific growth rates. Finally, we explain why the current age distribution, which is a product of past demographic history, can be expressed in terms of the *current* birth rate, mortality rates, and growth rates.

## Deriving a General Equation of Age Distribution

Let the density of a population at age $a$ and time $t$ be denoted by $N(a, t)$. Suppose that $N(a, t)$ is nonnegative and differentiable at $(a, t)$ for $0 < a < \omega$ and $-\infty < t < \infty$. It is assumed that $N(a, t) = 0$ for $a \geqq \omega$. Call $N(0, t) = B(t)$. Define

$$c(a, t) = N(a, t)/\int_0^{\omega} N(a, t)\, da.$$

In addition, $b(t)$ and $\mu(a, t)$ refer, respectively, to the birth rate at time $t$ and the instantaneous mortality rate at age $a$ and time $t$. Assuming that the population is closed to migration,[1] the age distribution of the population is given by (Preston and Coale, 1982):

$$c(a, t) = b(t) \exp\left[ - \int_0^a r(x, t)\, dx \right] p(a, t), \tag{1}$$

where

$$r(x, t) = [\partial \ln N(x, t)]/\partial t$$

and

$$p(a, t) = \exp\left[ -\int_0^a \mu(x, t)\, dx \right].$$

Multiplying both sides of equation (1) by the total population size gives

$$N(a, t) = B(t) \exp\left[ -\int_0^a r(x, t)\, dx \right] p(a, t). \tag{2}$$

We also introduce the cohort version of the period measure of mortality, $p(a, t)$, as follows:

$$s(a, t - a) = \exp\left[-\int_0^a \mu(x, t - a + x)\, dx\right].$$

$s(a, t - a)$ is thus the proportion surviving to age $a$ among the cohort of persons born at time $t - a$.

Let us proceed to develop an explicit expression for the age-specific growth rate. The size of the corhort aged $a$ at time $t$ is determined by its measure of past demographic experience as

$$N(a, t) = B(t - a)\, s(a, t - a). \tag{3}$$

Let $r_B(u)$ be the growth rate of the density of births at time $u$; that is,

$$r_B(u) = \frac{\partial}{\partial u} \ln B(u). \tag{4}$$

We also let $u = t - a + x$ so that

$$\begin{aligned} \frac{\partial}{\partial t} \ln s(a, t - a) &= -\int_0^a \frac{\partial \mu(x, t - a + x)}{\partial t}\, dx \\ &= -\int_0^a \left.\frac{\partial \mu(x, u)}{\partial t}\right|_{u=t-a+x} dx \\ &= -\int_0^a \left.\frac{\partial u}{\partial t}\frac{\partial \mu(x, u)}{\partial u}\right|_{u=t-a+x} dx \\ &= -\int_0^a \left.\frac{\partial \mu(x, u)}{\partial u}\right|_{u=t-a+x} dx, \end{aligned} \tag{5}$$

making use of the fact that $\partial u/\partial t = 1$. Now differentiating the natural logarithm of equation (3) with respect to $t$ and using equations (4) and (5), we have

$$r(a, t) = r_B(t - a) - \int_0^a \left.\frac{\partial \mu(x, u)}{\partial u}\right|_{u=t-a+x} dx. \tag{6}$$

Equation (6) is the basic equation needed to interpret age-specific growth rates.

Before proceeding to this interpretation, we show how the basic equations (1) and (2) can be derived from equation (3). The two terms on the right side of equation (3) are expressed as follows:

$$\begin{aligned} B(t - a) &= B(t) \exp\left[-\int_{t-a}^{t} r_B(u)\, du\right] \\ &= B(t) \exp\left[-\int_0^a r_B(t - x)\, dx\right], \end{aligned} \tag{7}$$

and

$$
\begin{aligned}
s(a,\ t-a) &= \exp\left[-\int_0^a \mu(x,\ t-a+x)\,dx\right] \\
&= \exp\left[-\int_0^a \mu(x,\ t)\,dx + \int_0^a \int_{t-a+x}^{t} \frac{\partial \mu(x,\ u)}{\partial u}\,du\,dx\right] \\
&= p(a,\ t)\exp\left[\int_0^a \int_0^x \frac{\partial \mu(y,\ t-x+y)}{\partial u}\,dy\,dx\right]. \qquad (8)
\end{aligned}
$$

The variable of integration changes between the second and third lines of equation (8). The second term within the brackets of the second line is a straightforward double integral. The partial derivative of the instantaneous age-specific mortality rate with respect to time $u$ is integrated first with respect to time $u$, then over age $x$, as shown in Figure 1A. With regard to the second term of the third line, on the other hand, the derivative of $\mu$ with respect to time $t$ is first integrated following the aging of the cohort born at $t - x$, as illustrated in Figure 1B.

By substituting equations (7) and (8) into equation (3), we obtain

$$
\begin{aligned}
N(a,\ t) &= B(t)\exp\left[-\int_0^a \left\{r_B(t-x) + \int_0^x \left.\frac{\partial \mu(y,\ u)}{\partial u}\right|_{u=t-x+y} dx\right\}\right] p(a,\ t) \\
&= B(t)\exp\left[-\int_0^a r(x,\ t)\,dx\right] p(a,\ t),
\end{aligned}
$$

making use of equation (6). Thus the use of an explicit expression for age-specific growth rates provides another way to derive the basic equation (2).

## Interpreting Age-Specific Growth Rate Functions

Equation (6) indicates that the age-specific growth rate, $r(a,\ t)$, is a function of two terms: One is the growth rate in the number (strictly speaking, density) of births $t - a$ years

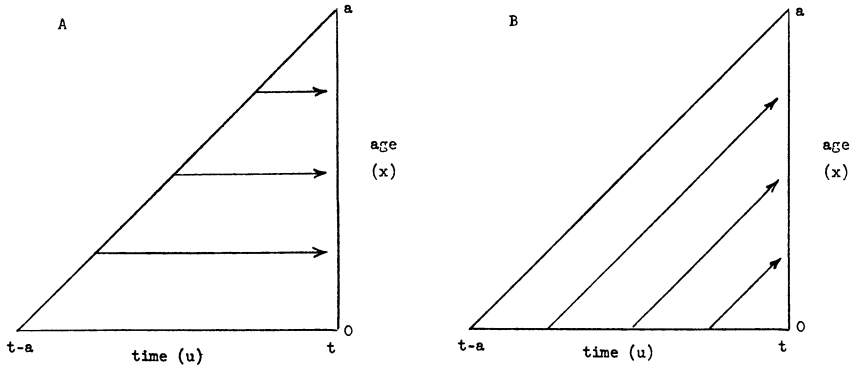

Figure 1. Change of Variable of Integration in Equation (8) With Respect to Time (A) and Following Cohort Experience (B)

earlier; the second is the cumulation of *changes* in age-specific mortality rates over the lifetime of the cohort aged *a* at time *t*, relative to the immediately preceding cohort. Age-specific growth rates thus do not depend on the *levels* of mortality in a cohort but, rather, on mortality rate *changes* that have been experienced by that cohort. If there have been no cross-cohort changes in mortality conditions for any living cohort, and if the growth rate of births has been constant, then the population is "stable:" there is no variance in age-specific growth rates, and equation (1) reduces to the familiar equation of a stable population. These would seem to be the simplest set of fertility and mortality conditions sufficient to produce a stable population. The equations of a stable population could still obtain in a population with changing demographic rates if the changes in the growth rate of births and those in mortality offset each other in a manner made explicit by equation (6).

The set of age-specific growth rates prevailing in a particular population can thus be highly instructive about the demographic history of the population. It is well known that changes in age-specific death rates tend to be largest at the very young and very old ages. Such a tendency is clearly reflected in empirically based model life table systems (e.g., Coale, Demeny, and Vaughan, 1983; United Nations Secretariat, 1982). In the early stages of a mortality decline, then, one would expect to find a U-shaped pattern of age-specific growth rates, ceteris paribus.

To demonstrate how actual patterns might look, we have simulated a variety of mortality declines that start from conditions of a stable population. All begin with a West female model life table having a life expectancy at birth of 40 years. The total fertility rate is constant at 5.25 children and the mean age at childbearing is constant at 29. The population is assumed to be closed to migration. This combination of assumptions produces a stable population with an annual growth rate for each age group of 0.0154.

Figure 2 shows the consequences for age-specific growth rates of imposing a continuous

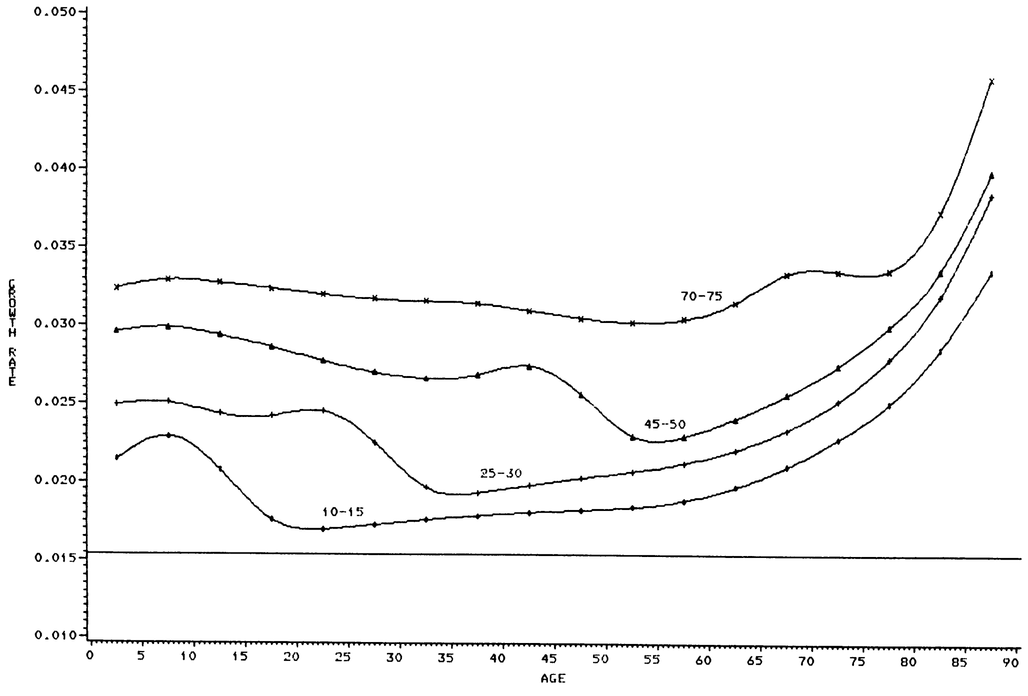

Figure 2. Age-Specific Growth Rates by Years Since Mortality Decline Began: Moderate Decline

mortality decline at a moderate pace of 0.5 years of life expectancy gained per calendar year in the West model life table system. Over time, the age-specific growth rate rises at every age. However, the age-profile of growth rates changes appreciably. Growth rates in the period 10–15 years after the decline begins are decidedly U shaped, with the exception of a slight rise from the youngest to the next youngest age. (This rise results from the declining mortality experienced by the 5- to 9-year-olds, including the large declines below the age of 5, for a period that is roughly three times longer than the period of decline for those who are 0–4 years old.) The right arm of the age-specific growth rate profile is larger than the left arm, suggesting that the full effects of the mortality decline cannot be readily simulated by declines in fertility (Coale, Demeny, and Vaughan, 1983). That cohort of persons who were approximately 7.5 years old at the time the mortality decline began have the lowest growth rate, a pattern that persists over time (as shown in Fig. 2). [This approximate figure of 7.5 was obtained by locating the growth rate of the age group ($a$, $a + 5$) in the period ($t$, $t + 5$) at age $a + 2.5$ and time $t + 2.5$.] Obviously, it reflects the fact that this cohort experienced neither the large mortality declines in early childhood nor, for many decades, the large declines at the older age. For at least a half-century after the decline begins, the length of the decline can be identified (in these idealized data) by subtracting about 7.5 years from the trough in the age-specific growth rate profile.

The curve to the left of this cohort-specific trough stretches out and becomes relatively level as time advances, reflecting two offsetting tendencies. As the mortality decline proceeds, decrements in age-specific death rates become smaller per unit of time, a condition that would by itself cause a rise in the profile from left to right in Figure 2. The growth rate in the number of births, however, increases over time as more rapidly growing cohorts reach the childbearing ages. By itself this tendency would cause the age-specific growth rate profile to fall from left to right. There is no reason why in the short run, these two tendencies must offset one another; but empirically, they tend to do so in this example. Eventually, the capacity for mortality decline in the ages before and during reproduction becomes exhausted

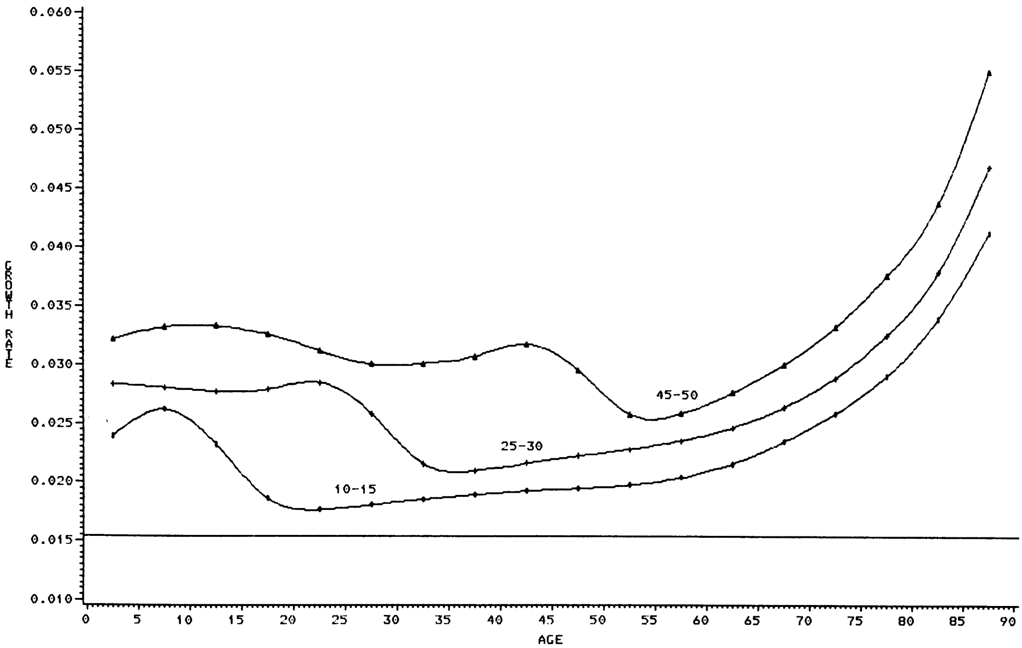

Figure 3. Age-Specific Growth Rates by Years Since Mortality Decline Began: Fast Decline

as death rates reach such low levels that further declines become extremely small. Therefore, the growth rate curve must become horizontal before the age of 50, assuming that fertility remains constant.

The right arm of the age-specific growth rate profile remains sharply upwardly sloped even 70–75 years after the mortality decline begins, by which time the population has achieved a life expectancy at birth of 77.5 years. In the first 50 years after the decline begins, the age-specific growth rates above the age of 55 have a highly regular, concave-upward pattern that is nearly parallel from period to period. The growth rate rises by about 0.017 from the ages of 55–59 to the ages of 85 and above. Although observed age-specific growth rates at these ages are subject to changes in the patterns of age misreporting, which is often serious in these ages, they are also capable of conveying a good deal of information about the recent pace of mortality decline.

Figure 3 shows equivalent information when mortality has declined more rapidly, at a rate of 0.75 years gained in life expectancy per calendar year (still maintaining the assumption of constant fertility and no migration). This is roughly the average pace of mortality change in China between 1950 and 1980, though the course was highly irregular (Coale, 1983). The age pattern of change is very similar to that pertaining to the moderate pace of decline, as can be verified by superimposing Figure 2 on Figure 3. The vertical scales of these figures differ, suggesting that patterns of change can be accurately expressed as simple linear functions of one another, with the parameters depending on the pace of mortality change. The right arm of Figure 3 is steeper than that of Figure 2, rising by about 0.023 at the ages of 55–59 to 0.029 at the ages of 85 and above.

Figure 4 shows the pattern of age-specific growth rates when mortality declines at the rapid pace just assumed but stops declining altogether after 30 years, by which point life expectancy has reached 62.5 years. Beyond 30 years of the projection period, the age-specific

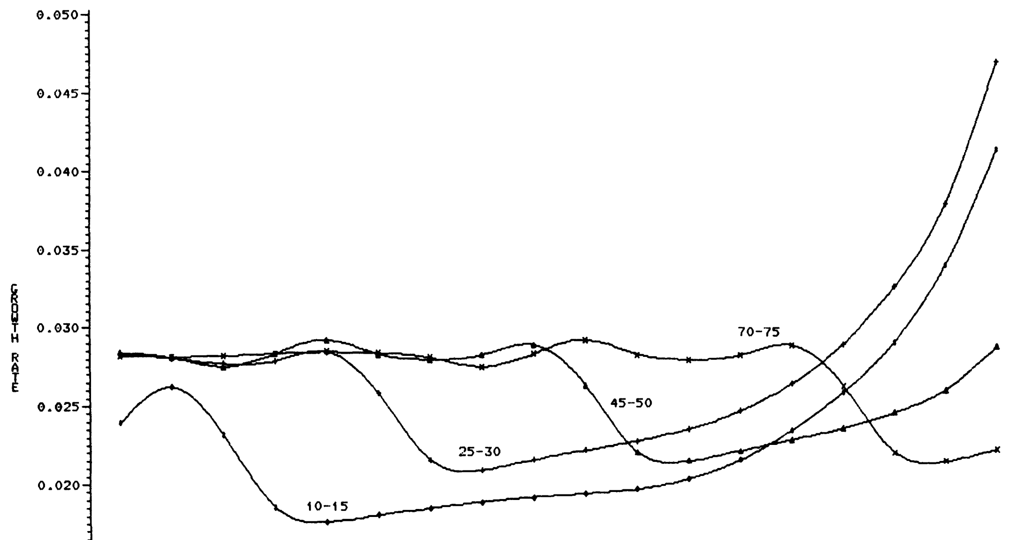

Figure 4. Age-Specific Growth Rates by Years Since Projection Began: Constant Fertility, 0.75-Year Increase in Life Expectancy per Year, Stops After 30 Years

growth profile simply shifts to the right by 1 year of age for every 1 year of calendar time. Since there is no subsequent mortality change for a cohort after 30 years, there can be no other source of change in the cohort's age-specific growth rate. Thus the age pattern of growth rates begins to move along in a strictly cohort-specific fashion as soon as the mortality decline ceases.

The effects of fertility changes on age-specific growth rates are more straightforward. In the first year that fertility begins to decline in a previously stable population, the population below the age of 1 grows more slowly than the population at other ages. After 15 years of decline, the population at all ages below 15 is growing more slowly. In the period beyond 15 years, the declines occurring in age-specific fertility are reinforced for the infant population by slower growth in the population of reproductive ages. These tendencies are illustrated in Figure 5: Life expectancy is assumed to be constant at a value of 40 years and age-specific fertility rates to decline at a constant rate of 1.5 percent per year from initially stable values that produced a total fertility rate of 5.25. In the initial years of the fertility decline, the population of reproductive age is growing at about 1.54 percent per year, which offsets the annual decline in age-specific fertility rates of 1.5 percent and produces a brief period of nearly zero growth in births. As these zero-growth cohorts enter the reproductive ages, the absolute number of births begins to decline and age-specific growth rates turn negative.[2]

## Examples of Selected Empirical Age-Specific Growth Profiles

In this section we show several examples of empirical growth rate patterns. These are observed over 10- or 20-year periods, and the correspondence to the 5-year periods displayed earlier is not exact. The rates observed over longer periods are, however, simply unweighted averages of the rates observed over individual 5-year periods.

Figure 6 presents age-specific growth rate profiles for Indian males over extended periods during the 20th century.[3] The 1891–1911 rates are basically level with age, although

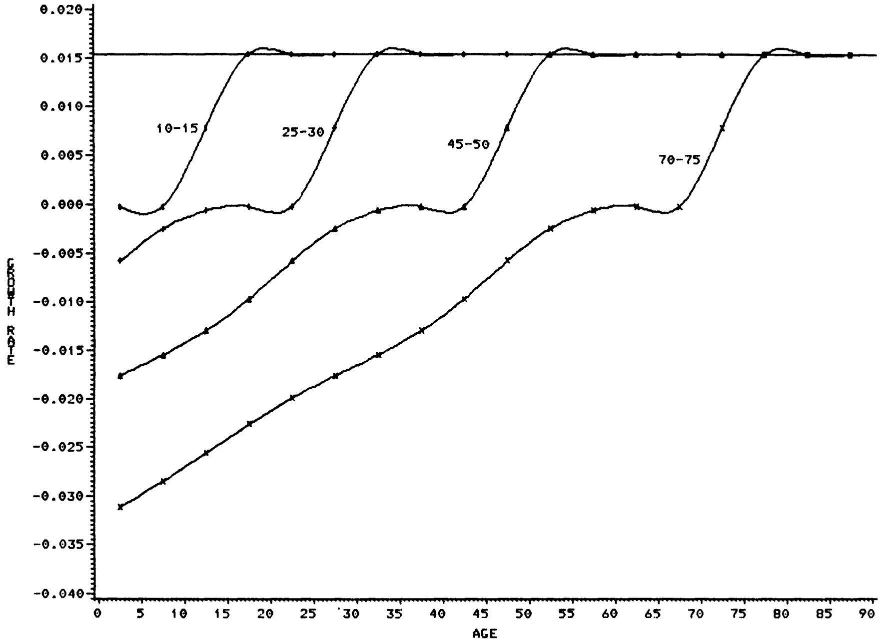

Figure 5. Age-Specific Growth Rates by Years Since Projection Began: Constant Mortality, Life Expectancy of 40, 1.5 Percent Decline in TFR per Year

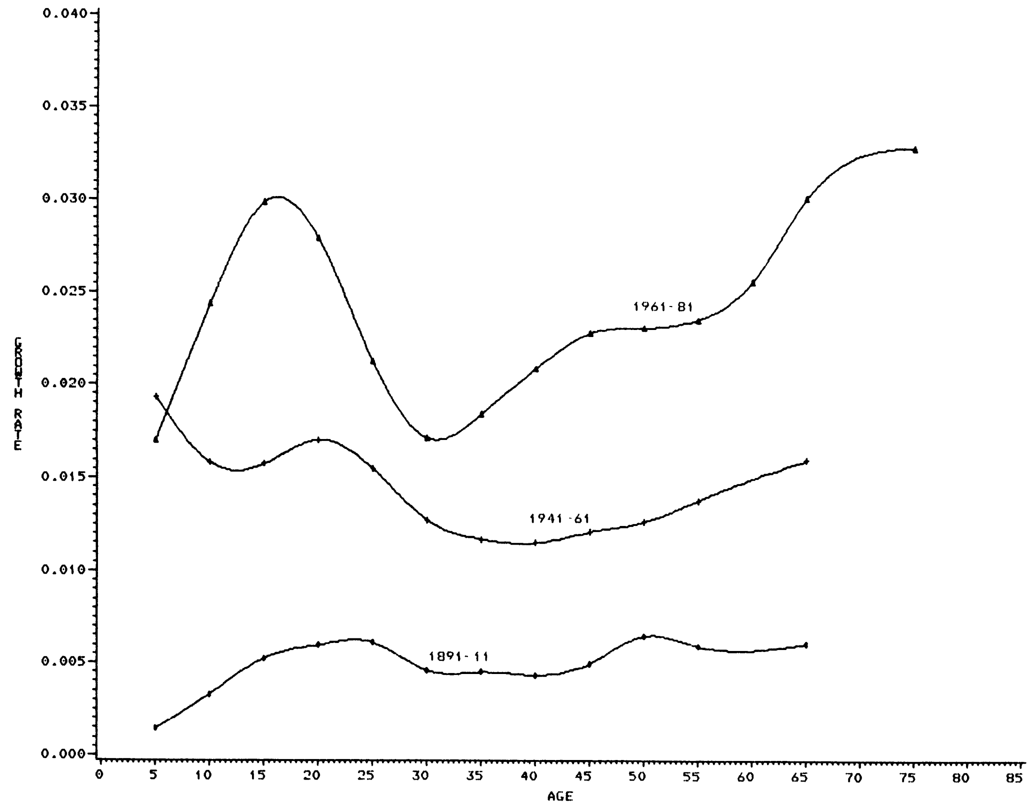

Figure 6. Age-Specific Growth Rates: India, Males

the unsystematic variation that is clearly present may reflect changes in the quality of age reporting. The profile in 1941–1961 is higher at every age and U shaped, consistent with a history of mortality decline. The pattern for 1961–1981 is sharply U shaped beyond the ages of 15–20. The trough at age 30 is indicative of a period of accelerated mortality decline that may have begun around 1949 (1971 − 30 + 7.5). This agrees with the estimated mortality trends from other sources. According to the estimates officially adopted by the government of India, the expectation of life at birth for males was 32.1 for 1931–1941, 32.4 for 1941–1951, and 41.9 for 1951–1961 (Government of India, undated:53). The lack of improvement from the 1930s to the 1940s followed by a substantial rise from the 1940s to the 1950s is consistent with a turning point somewhere around 1950. In the 1961–1981 graph, the upward slope below the age of 15 is consistent with either declining fertility or a deceleration in the pace of mortality decline; the former explanation appears more consistent with other information (Bhat, Preston, and Dyson, 1984).

Figure 7 presents two age-specific growth rate profiles for two European countries in the past. The first two census age distributions in England and Wales are available for 1821 and 1841. The pattern of intercensal growth rates, taken from Lentzner (1985), is a remarkably smooth inverted U. Such a pattern would be produced by *rising* mortality in the early stages of the British industrial revolution. This possibility is also suggested by declining standards of nutrition and sanitation and increased urbanization in the early 19th century (Woods and Woodward, 1984). Elaborate reconstructions of British demography do not support this possibility, although they leave ample room for debate (Wrigley and Schofield, 1981:chap. 7). The point is that intercensal population change by age can be used to describe not only

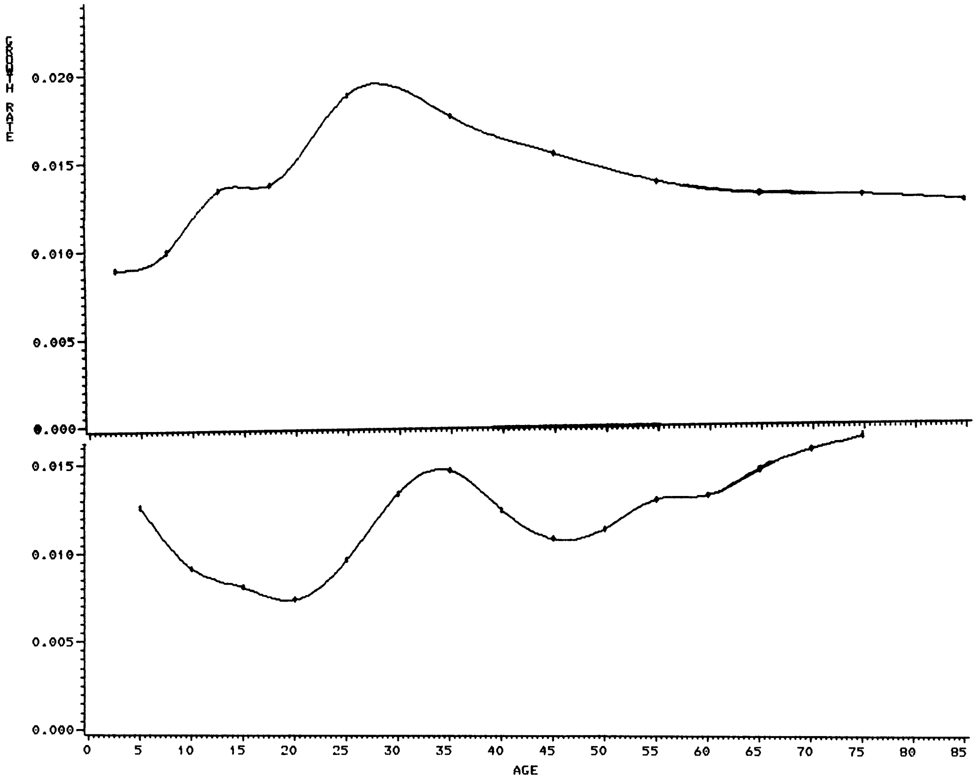

Figure 7. Age-Specific Growth Rates: Females in (top) England and Wales (1821–1841) and (bottom) Portugal (1920–1940)

the *level* of mortality but also its previous *trend*. In this case, it provides an important new piece of evidence for an old debate.

Mortality decline came to southern Europe later than to northern and western Europe, but once it began it proceeded more rapidly (Stolnitz, 1956). Life expectancy at birth for the Portuguese females rose from 38.7 years in 1920 to 53.2 years in 1940 (Preston, Keyfitz, and Schoen, 1972). The pattern of age-specific growth for females during 1920–1940 is shown in Figure 7. The basic pattern is a smooth U shape with a trough at age 20, suggesting a duration of mortality decline of about 12.5 years in 1930, or one that began during or just after World War I (and its influenza epidemic). Superimposed on this U-shaped curve is a smaller hill-shaped pattern peaking at age 35. It is plausible that this secondary pattern reflects a reduction in rates of net out-migration during 1920–1940 from their historic levels—perhaps a result of the imposition of strict U.S. immigration quotas for persons from southern Europe in the 1920s.

Age-specific growth rate profiles cannot be unambiguously interpreted. The examples just presented are unusually clear. The profiles in many developing countries are particularly vulnerable to changes in the quality of age reporting and the completeness of censuses. Among other things, such changes can obscure the right arm of the profile when mortality is declining because overstatement of age among the elderly is likely to be reduced over time. Since these profiles are an integral part of current demographic accounts, they may merit the same kind of effort in reconstructing their true values as has been devoted to birth rates, death rates, and age distributions.

## Discussion

The age distribution equation (1) gives rise to an important question: Why can the *current* age distribution, which is a product of *past* history, be determined solely by the *current* rates? The derivation given earlier in this article provides an intuitively straightforward and demographically meaningful answer to the question. This is not necessarily the case for the derivation given in previous literature (Bennett and Horiuchi, 1981; Preston and Coale, 1982). In the new derivation, we move back and forth on the time scale. The previous derivation, however, is an algebraic manipulation of relationships among derivatives of demographic variables at a *fixed* point of time. The substantive meanings of the new derivation can be more clearly understood by comparing the actual age distribution and the implied stationary age distribution.

The density function of the stationary population that is fashioned out of current births and mortality schedules is given by

$$n(a, t) = B(t)p(a, t), \tag{9}$$

where $n(a, t)$ is the size that the cohort aged $a$ at time $t$ would attain if the cohort had the size $B(t)$ at birth and had mortality experiences as observed at time $t$.

The logarithm of the ratio of the current size of the actual cohort at age $a$ to the size of the synthetic cohort is simply

$$\ln \frac{N(a, t)}{n(a, t)} = -\int_0^a r(x, t)\, dx, \tag{10}$$

from equation (2). Equation (10) implies that the current age distribution can be obtained by deriving the underlying stationary distribution and converting it into the actual distribution by using current age-specific growth rates. The initial question of why the current age distribution can be solely determined by current demographic rates can be rephrased in a more specific way: Why does the integral of the age-specific growth rate function indicate differences between the actual and stationary age distributions?

Let us focus on the cohort currently aged 40 as an example. Its size is denoted by $N(40, t)$, and the size of the corresponding synthetic cohort is $n(40, t)$. The actual cohort started with a cohort size at birth, $B(t - 40)$, and was subjected to a sequence of age-specific mortality rates, including, for example, $\mu(10, t - 30)$, $\mu(20, t - 20)$, $\mu(30, t - 10)$, and $\mu(40, t)$. Corresponding elements of the demographic life history of the synthetic cohort are $B(t)$, $\mu(10, t)$, $\mu(20, t)$, $\mu(30, t)$, and $\mu(40, t)$.

Suppose that the difference between $N(40, t)$ and $n(40, t)$ is partly attributable to the difference in mortality at the age of 10 between 30 years ago and now. The actual cohort currently aged 40 experienced a mortality rate of $\mu(10, t - 30)$. The synthetic cohort is assumed to have experienced the current mortality rate at the age of 10, that is, $\mu(10, t)$. If mortality has been declining during the last 30 years, $\mu(10, t)$ is lower than $\mu(10, t - 30)$. This raises $n(40, t)$ relative to $N(40, t)$. Suppose that the decline was particularly sharp 15 years ago. The sudden fall of the mortality rate at the age of 10 immediately increased the population size at the age of 10 for cohorts reaching that age after the decline, relative to that of cohorts passing that age before the decline. This produced a high value of $r(10, t - 15)$. The increase in population size at the age of 10 that occurred 15 years ago is now echoed as a population growth at age 25, resulting in a high value of $r(25, t)$. In this way, a past mortality change that produced the difference in size between the actual and synthetic cohorts currently aged 40 is reflected in the current growth rate at the age of 25. To generalize, a change in the number of births or a change in the mortality rate at age $x$ that occurred $u$ years ago contributes to the difference between $N(a, t)$ and $n(a, t)$ and is included in $r(u + x, t)$, where $x = 0$ for births and $u + x \leq a$.

In sum, the relative difference in size between the actual and synthetic cohorts aged $a$ on the left side of equation (10) can be decomposed into relative differences in the determinants of cohort size (i.e., the number of births and the age-specific mortality rates) between the two cohorts. The differences result from changes over time in those determinants, as shown in equations (7) and (8). Those changes were experienced by cohorts younger than $a$ and are therefore contained in the current growth rates of those cohorts, as in equation (6). This explains why all differences between the actual age distribution and the implied stationary age distribution are embedded in the integral of the age-specific growth rate function and, in turn, why the current age distribution can be solely determined by current demographic rates.

Although applications of the age-distribution equation have so far been heavily directed toward making estimates from incomplete data, our results suggest the usefulness of age-specific growth rates in empirical studies of age structure. By incorporating age-specific growth rates in the analysis, changes in the age distribution of a population can be better understood in terms of mortality and fertility levels of the population (e.g., see Preston, 1986; United Nations Secretariat, in press). Age-specific growth rates also provide useful clues about the demographic history of a population in periods before the rates were recorded. They deserve a place among the normal objects of demographic inquiry.

## Notes

[1] Most equations in this article can be easily generalized to populations open to migration by incorporating age-specific rates of net migration in the same way that age-specific mortality rates are incorporated (Preston and Coale, 1982). It should be noted, however, that those two kinds of rates have very different characteristics. The rate of net migration is the difference between the in-migration and out-migration rates. The in-migration rate cannot be interpreted as a measure of a risk to which members of a population are exposed, since it is only members of other populations who are at risk of migrating into the population in question.

[2] The simulation results clearly depend on the specified initial condition and assumed patterns of change. The simulation started from a stable population, in which growth rates are constant for all ages. Then effects of mortality and fertility changes on age-specific growth rates were superimposed on the cohort sequence of growth rates that had existed before those changes started. The initial sequence is not always level, because famine, war, epidemic, or some other event may produce substantial fluctuations in vital rates and, in turn, age-specific growth rates. By assuming a level sequence at the onset of simulation, however, we have illustrated some *typical* effects of mortality decline and fertility decline on age-specific growth rates.

[3] The rates were computed by P. N. Mari Bhat after adjustment for intercensal boundary change.

## Acknowledgments

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