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ASSESSING THE EFFECTS OF MORTALITY REDUCTION ON POPULATION AGEING*

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SUMMARY

This article presents a new method for decomposing age distribution changes into changes in the number of births and changes in age-specific rates of mortality and migration. The method is developed on the basis of the equation for the age-specific growth rate proposed by Horiuchi and Preston (1988). Using this method, it is shown that the increase in the proportion of old women in Japan during 1970-1980 is mainly due to the reduction of mortality, particularly at old ages. The results lend support to the proposed idea that the pattern of age structure changes in developed countries is now shifting from fertility-dominated to mortality-dominated ageing.

Reduction in fertility and mortality leads to the ageing of populations. Fertility decline reduces the proportion of young children. Although the improvement of survival chances at a high mortality level may at first make the age distribution younger by reducing child mortality substantially, it will eventually contribute to population ageing by increasing the proportion of the population who reach very old ages.

Previous studies of the impact of fertility and mortality on population ageing have shown that the impact of fertility is significantly greater than that of mortality (Coale, 1956 and 1957; United Nations, 1956). The major focus of those studies seems to be on population dynamics generally observed in the course of the demographic transition: changes in fertility from an uncontrolled, high level to the neighbourhood of the replacement level (about two children per woman), and changes in the mortality level accompanying the epidemiologic transition of major causes of death from infectious and parasitic diseases to degenerative diseases.

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Most developed countries these days, however, have completed the demographic transition and entered into new phases. Emerging characteristics of nuptiality, fertility and family formation in those populations have been discussed under the label of "the second demographic transition" (van de Kaa, 1987). Concerning mortality and morbidity, those populations passed the three stages of epidemiologic transition (Omran, 1971), and entered a new, fourth stage of "delayed degenerative diseases" (Olshansky and Ault, 1986) or the "hybristic" stage, in which mortality is increasingly influenced by individual behaviour and lifestyles (Rogers and Hackenberg, 1987).

Population dynamics in the new phase are different from those during the demographic transition. Yu and Horiuchi (1987) analyse the effects of fertility and mortality changes on growth rates of different age groups and state that the importance of the effect of mortality on population ageing, relative to that of fertility, is increasing in developed countries (United Nations, 1988). They indicate the following three reasons for the rising significance of mortality.

First, the fertility decline tends to stop or slow down after reaching the neighbourhood of the replacement level, which is the total fertility rate (TFR) of about 2.1. Sometimes the decline may continue within the below-replacement zone. But even so, the decline of fertility below the replacement level is not as fast as the decline during the demographic transition, in which the TFR may fall from six or more children per woman to about two children per woman. Mortality, on the other hand, has been declining without a significant slow-down in many countries, passing through upper limits of life expectancies assumed in earlier projections (see, for example, Bourgeois-Pichat, 1978).

Secondly, significant mortality reduction in those populations can be achieved only at old ages. In general, mortality decline contributes to the growth of population of all age groups. This feature of mortality decline usually dilutes its effects on the age distribution, because the age composition remains constant if all age groups grow at the same rate. However, as death rates at young ages becomes negligibly small, room for significant mortality improvement is left only for old ages. Moreover, old-age mortality due to degenerative diseases is now on a substantial decline.

Finally, the population ageing itself strengthens mortality effects on the age structure. Although fertility effects on population growth are pronounced at young ages, mortality effects tend to be stronger at older ages. However, when the proportion of population at old ages is very small, even a large proportional growth of the age group may change the entire age distribution only slightly. An increasing proportion of the elderly population amplifies the impact of mortality reduction on the ageing of the entire age structure.

Taking into consideration the rising importance of mortality improvement to population ageing, it seems useful to distinguish four stages in a typical course of age structure changes. The first stage is characterized by stable, young age structures before the demographic transition. The continuation of uncontrolled fertility and high mortality keeps the age distribution young, which may be well-approximated by a stationary population model or a low-growth stable population model, except for perturbation due to catastrophic events such as famines and epidemics.

Then comes the stage of the increasingly younger age distribution. In a typical sequence of demographic transition, the initiation of significant mortality decline precedes that of fertility decline. The reduction from very high levels of infant and child mortality raises the population growth rate, and increases the number of young children in particular, thereby making the already young-age distribution even younger. In addition, such factors as decline in sterility, reduction in miscarriages, shortened periods of post-partum amenorrhoea and reduced widowhood raise the high fertility level further, thereby leading to the younger age structure.

The transition from the second to the third stage occurs when a significant decline of fertility starts. Falling fertility causes a continuous decrease in the proportion of young children, thereby making the age distribution older. This stage, therefore, should be called the stage of fertility-dominated population ageing. The turning point of world population growth around 1970 from the rising growth rate in the 1950s and 1960s to the falling growth rate in the 1970s, caused by the initiation of substantial fertility decline in a number of countries in Asia and Latin America, was also a turning point of age structure from the decreasing median age to the increasing median age (United Nations Secretariat, 1987).

Developed countries these days are entering into a new, fourth stage of mortality-dominated population ageing. After the completion of the demographic transition, fertility remains relatively stable, and mortality improvement, now concentrating in degenerative disease mortality at old ages, gradually becomes the major driving force of population ageing. Fertility in some populations may continue to fall further into the below-replacement zone, but the speed of decline is considerably slower than the speed in the previous stage.

On the basis of the multistage model of age structure changes described above, relationships between changes in age-specific mortality rates and the number and proportion of old women in Japan are analysed in this article. The female population in Japan, with the highest expectation of life at birth, is considered to be a front runner in the transition from the third to the fourth stage of age structure changes. A new method of assessing the impact of mortality changes at different ages on the age structure is developed for this study as an application of the age-specific growth rate equation.

METHOD

Horiuchi and Preston (1988) have shown that the growth rate of population at age a and time t, $r(a,t)$, can be decomposed into the growth rate of births, cumulated changes in age-specific death rates and cumulated changes in the age-specific rates of net migration that the cohort experienced (see also Preston and Coale, 1982, footnote 2). Let $r_B(t)$ be the growth rate of the number of births at time t and $\mu(a,t)$ and $m(a,t)$ be the instantaneous age-specific death rate and instantaneous rate of net-migration, respectively, at age a and time t. We have

$$r(a,t) = r_B(t-a) - \int_0^a \frac{\partial \mu(x,u)}{\partial u}\,dx + \int_0^a \frac{\partial m(x,u)}{\partial u}\,dx, \tag{1}$$

where the derivatives are assessed at $u = t - a + x$. It should be noted that equation (1) makes it possible not only to decompose the growth rate into birth effects, mortality effects and migration effects but also to decompose the total mortality effects into age-specific mortality effects and even into the age-and-cause-specific mortality effects if data on deaths by age and cause are available.

It should also be realized that the use of equation (1) for estimating the impact of mortality on population growth does not necessarily require mortality data. If migration is not significant or migration estimates are available for the cohorts to be studied, data on the size of those cohorts at their same ages will suffice. Such a data set makes it possible to derive effects of mortality changes in those age intervals. This is understood by comparing equation (1) with the following equation:

$$r(a,t) = r_B(t-a) + \int_0^a \frac{\partial}{\partial x} r(x, t-a+x)dx.$$

For estimating the effects of mortality, it is only necessary to follow the history of the study cohorts in terms of changes in age-specific growth rates. Data on the number of births in the remote past may not be required either, unless the interest is in deriving the effects on the current elderly population of improvements of child mortality that occurred a long time ago when they were very young, separately from the effects of the past increase of births. If the youngest age for which population data on the study cohorts are available is, for example, 30, then the combined effects on population growth of changes in the number of births and changes in mortality from birth to age 30 will be obtained.

Since the migration term in equation (1) can be handled in the same way as the mortality term, it is hereafter assumed that the population is closed to migration, in order to avoid unnecessary complexities. In the actual data analysis described later, the migration component is included, treating the re-annexation of Okinawa in 1972 as a special type of migration.

The absolute increase of the population aged a can be decomposed by multiplying equation (1) by $N(a,t)$, the density of population size at age a and time t. Then, by integrating it from age a to age b, decomposition results of the age group a to b are obtained. An alternative way to decompose the absolute increase is to differentiate the following equation with respect to t:

$$N(a,t) = B(t-a) - \int_0^a D(x, t-a+x)dx,$$

where $B(t)$ is the density of births at time t and $D(a,t)$ is the density of deaths at age a and time t. However, this alternative approach does not meet the purpose of the present study, in which the focus is not on changes in the number of deaths but on changes in the risk of death.

Equation (1) leads to an expression for the growth rate of total population. Let $c(a,t)$ be $N(a,t)$ divided by the total population size at t. Now the growth rate of total population at t is given by

$$r_T(t) = \int_{-\infty}^{t} c(t-u,t)\, r_B(u)du - \int_0^{\infty}\int_{-\infty}^{t} c(a+t-u,t)\frac{\partial\mu(a,u)}{\partial u}du\, da \quad (2)$$

The growth rate of total population is the sum of a weighted mean of past growth rates in the number of births and a weighted sum of past changes in age-specific death rates. (A more detailed discussion of equation (2) is given in annex II.)

Changes in the proportion of population aged a can be decomposed by substituting equations (1) and (2) into the following equation (3):

$$\frac{\partial c(a,t)}{\partial t} = c(a,t)\,[r(a,t) - r_T(t)]. \quad (3)$$

This makes it possible to assess the direction and intensity of the effects of changes in age-specific mortality rates on the proportion of population in the given age group. A discrete version of the above formulations is used in actual data analysis (see annex I).

Another way to decompose changes in the age distribution into fertility effects and mortality effects is the method of comparative population projections (United Nations, 1956 and 1988). An advantage of the present method in comparison with the method of comparative projections is the fact that the former method makes it easy to assess the effect of mortality at different ages. Although total mortality effects can be assessed using the method of comparative population projections, it is very difficult to use it to assess age-specific mortality effects.

It is important to distinguish direct and indirect effects of mortality changes on population growth. The present method is concerned with direct mortality effects only. Mortality reduction, however, has some indirect effects. It raises the chances of survival from birth to ages of child-bearing, thereby increasing the number of births and, in turn, the number of young children. Such indirect mortality effects working through fertility are difficult to assess when the present method is adopted. Mortality effects computed by using the method of comparative population projections, on the other hand, contain both direct and indirect effects.

Another important dimension is the distinction between period-oriented approaches and cohort-oriented approaches. Although the focus of the method of comparative population projections is on mortality changes in given periods, the present method is concerned with mortality changes between cohorts. Another method for assessing the impact of changes in period mortality on the age distribution has been developed by Takahashi (1986).

Data

Two sets of demographic estimates available in machine-readable form greatly enhance the opportunities for detailed analysis of recent demographic history in Japan. First, the demographic history of Japan from 1947 to 1984 has been reconstructed for each calendar year and each single-year age group by the Institute of Population Problems (1985). Secondly, Kobayashi and Nanjo (1988) have produced a set of annual complete life-tables from 1891 to 1986 and rearranged them by cohort.

This study follows cohorts retrospectively back to 1947, using the estimates of the Institute of Population Problems, then follows them further in the past by reverse-surviving from 1947, using the cohort life-tables by Kobayashi and Nanjo.

It should be noted, however, that such an elaborated data set is not a necessary condition for using the present method. A series of censuses 10 years apart, with five-year age groups, make possible an application, although in a less elaborated manner, of this method.

Results

The increments of the number of women aged 60, 70 and 80 (last birthday) in each of the three decades from 1950 to 1980 are decomposed in table 1. The decomposition is based on equation (1) multiplied by $N(a,t)$. Changes in 10 years beginning on 1 January of the first year of the decade are analysed. The youngest age interval for which mortality effects are estimated is determined by the availability of population estimates of the oldest study cohort at their young ages. Migration effects are derived by dealing with the re-annexation of Okinawa in 1972 as a special kind of international migration. Other types of international migration are assumed to be negligibly small.

The old-age female population in Japan grew rapidly in the recent past. The number of women aged 60 almost doubled in three decades, from 271,413 in 1950 to 502,529 in 1980. Out of the increment of 231,116, 18 per cent is attributable to the decline in mortality at ages 50-60. Changes in mortality at ages 10-60 explain 54 per cent of the growth of the 60-year-old female population, and the rest of the increase, which is 46 per cent, is due to changes in childhood mortality under age 10 and changes in the number of births.

The population growth rate in Japan tends to be higher at older ages. The number of women aged 70 more than doubled during the same 30-year period, from 169,166 in 1950 to 386,397 in 1980. Twenty-seven per cent of the growth was produced by mortality reduction in ages 60-70 and 12 per cent by mortality reduction in ages 50-60.

The proportional growth in the number of women aged 80 is even greater. The number of 80-year old women tripled from 49,027 in 1950 to 165,409 in 1980. Forty-four per cent of the increase is explained solely by the mortality improvement from age 70 to 80, compared with 25 per cent jointly explained by the mortality reduction under age 30 and the growth in the number of births.

TABLE 1. DECOMPOSITION OF INCREASE OF WOMEN AGED 60, 70 AND 80, JAPAN, 1950-1980

Factor	1950-1960 Increase	1950-1960 Percentage	1950-1960 Growth rate	1960-1970 Increase	1960-1970 Percentage	1960-1970 Growth rate	1970-1980 Increase	1970-1980 Percentage	1970-1980 Growth rate
Women aged 60									
Total increase	43 863	(100.0)	1.5	122 876	(100.0)	3.3	64 377	(100.0)	1.4
Mortality 50-60	17 173	(39.2)	0.6	12 569	(10.2)	0.4	11 574	(18.0)	0.3
Mortality 40-50	3 911	(8.9)	0.1	15 546	(12.7)	0.5	8 952	(13.9)	0.2
Mortality 30-40	4 535	(10.3)	0.2	9 114	(7.4)	0.3	14 788	(23.0)	0.3
Mortality 20-30	18	(0.0)	0.0	5 870	(4.8)	0.2	9 581	(14.9)	0.2
Mortality 10-20	1 430	(3.3)	0.1	1 785	(1.5)	0.1	4 117	(6.4)	0.1
Mortality 0-10 and births	16 796	(38.3)	0.6	77 991	(63.5)	2.4	11 263	(17.5)	0.2
Migration (re-annexation)	—			—			4 102	(6.4)	0.1
Women aged 70									
Total increase	42 072	(100.0)	2.2	48 581	(100.0)	2.3	126 578	(100.0)	4.0
Mortality 60-70	22 675	(53.9)	1.3	14 443	(29.7)	0.7	22 123	(17.5)	0.8
Mortality 50-60	1 697	(4.0)	0.1	13 366	(27.5)	0.6	10 358	(8.2)	0.4
Mortality 40-50	2 573	(6.1)	0.2	3 044	(6.3)	0.1	12 812	(10.1)	0.5
Mortality 30-40	725	(1.7)	0.0	3 530	(7.3)	0.2	7 511	(5.9)	0.3
Mortality 20-30	1 397	(3.3)	0.1	14	(0.0)	0.0	4 837	(3.8)	0.2
Mortality 0-20 and births	13 005	(30.9)	0.7	14 185	(29.2)	0.7	65 743	(51.9)	2.5
Migration (re-annexation)	—			—			3 193	(2.5)	0.1
Women aged 80									
Total increase	33 564	(100.0)	5.2	33 996	(100.0)	3.4	48 822	(100.0)	3.5
Mortality 70-80	17 014	(50.7)	3.0	13 455	(39.6)	1.5	20 770	(42.5)	1.7
Mortality 60-70	402	(1.2)	0.1	11 071	(32.6)	1.3	7 971	(16.3)	0.7
Mortality 50-60	1 094	(3.3)	0.2	828	(2.4)	0.1	7 377	(15.1)	0.6
Mortality 40-50	166	(0.5)	0.0	1 256	(3.7)	0.2	1 680	(3.4)	0.1
Mortality 30-40	901	(2.7)	0.2	354	(1.0)	0.0	1 948	(4.0)	0.2
Mortality 0-30 and births	13 987	(41.7)	2.5	7 031	(20.7)	0.8	7 837	(16.1)	0.7
Migration (re-annexation)	—			—			1 239	(2.5)	0.1

Results in table 1 indicate that the decline of adult mortality, particularly the recent improvement at old ages, has stronger impacts on the growth of the elderly female population in Japan than the growth in the number of births and the reduction of childhood mortality in the past when those old persons were born or were young children. This tendency is more pronounced for the growth of female population at older ages. These analyses, however, do not include the effects on population ageing of recent fertility decline in Japan. It is necessary, therefore, to proceed to decomposition of the increase in the proportion of all women who are old.

In table 2, changes in the proportion of women who are aged 60 or over, 70 or over, and 80 or over are decomposed into effects of old-age mortality (aged 60 and above); effects of middle-age mortality (aged 30 to 60); combined effects of young-age mortality (below age 30) and fertility (the number of births); and effects of the re-annexation of Okinawa.

TABLE 2. DECOMPOSITION OF INCREASE IN THE NUMBER AND PROPORTION OF WOMEN IN OLD AGES, JAPAN, 1970-1980

Factor	*Number of women*	*Percentage*	*Proportion of women percentage*	*Percentage*
		Aged 60 or over		
(1980)	8 448 542		14.35	
(1970)	6 038 553		11.52	
Total increase	2 409 989	(100.0)	2.83	(100.0)
Mortality 60 or over	642 233	(26.6)	0.96	(34.1)
Mortality 30-60	655 495	(27.2)	0.91	(32.3)
Mortality 0-30 and births	1 042 034	(43.2)	0.93	(32.9)
Migration (re-annexation)	70 228	(2.9)	0.02	(0.7)
		Aged 70 or over		
(1980)	3 823 983		6.49	
(1970)	2 515 988		4.80	
Total increase	1 307 995	(100.0)	1.69	(100.0)
Mortality 60 or over	547 539	(41.9)	0.88	(51.8)
Mortality 30-60	262 261	(20.1)	0.36	(21.3)
Mortality 0-30 and births	466 189	(35.6)	0.44	(26.1)
Migration (re-annexation)	32 006	(2.4)	0.01	(0.8)
		Aged 80 or over		
(1980)	1 010 848		1.72	
(1970)	628 215		1.20	
Total increase	382 633	(100.0)	0.52	(100.0)
Mortality 60 or over	265 896	(69.5)	0.44	(84.6)
Mortality 30-60	47 226	(12.3)	0.06	(11.4)
Mortality 0-30 and births	60 519	(15.8)	0.02	(3.0)
Migration (re-annexation)	8 992	(2.4)	0.00	(0.9)

Note: The number and the proportion of women at the beginning of respective years are shown. Women aged 100 or over are not included in those three age groups because of difficulties in deriving reliable estimates of the size of those cohorts when they were very young.

Women aged 100 and over are not included in the denominator and numerator of the proportion, because they are very few and reliable population estimates of those cohorts at their young ages are not available. Given the lack in table 1 of clear systematic trends within the 30-year period, it is decided that the focus of this analysis is placed on changes in the 1970s.

The proportion of women aged 60 or over increased from 11.5 per cent of all women in 1970 to 14.3 per cent in 1980. Contributions to the increment, which is 2.8 per cent of the total female population, are almost equally split among the three major factors—namely, old-age mortality, middle-age mortality, and the combination of fertility and young-age mortality. The relative importance of old-age mortality, however, rises with age. More than half of the increase of the proportion of women aged 70 or over is attributable to the decreasing old-age mortality. As for women aged 80 or over, 85 per cent of the increase of its proportion is explained by the reduction in old-age mortality.

In summary, results of the present study indicate that the ageing of female population in Japan in the recent past is mainly due to the decline of mortality, particularly the improved survival chances at old ages. These mortality effects tend to be more pronounced for the increase of the proportion in older age groups. These findings seem to provide quantitative evidence of the view that populations in developed countries are now moving from the stage of fertility-dominated ageing to the stage of mortality-dominated ageing. In addition, the method adopted in this study, which has been developed as an application of the age-specific growth rate equation by Horiuchi and Preston (1988), proved to be a powerful tool for analysing changes in the age structure.

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ANNEX I

Decomposition of changes in age distribution

This annex describes a procedure for applying to discrete data the methodology for decomposing changes in the age distribution.

Basic framework

The change in the proportion of population in a given age group between two time points will be decomposed. The two time points are denoted by t_1 and t_2. The age interval is from age a to b. The age in this annex is not the exact age but rather the age on the last birthday.

Let $N(y,t)$ be the number of persons age y at time t. The number of persons aged a to b, the number of persons at all ages, and the proportion of population aged a to b are given by:

$$G_j = \sum_{y=a}^{b} N(y, t_j)$$

$$S_j = \sum_{y=o}^{\infty} N(y, t_j)$$

and

$$P_j = G_j/S_j,$$

respectively, for time t_j, where $j = 1$ or 2.

In addition, let

$$g_j(y) = N(y, t_j)/G_j$$

and

$$s_j(y) = N(y, t_j)/S_j$$

for later use.

In what follows, the change in the proportion of population aged a to b (i.e., $P_2 - P_1$) will be decomposed into contributions of past population dynamics that occurred in different age intervals. Those age intervals are set up by selecting certain ages $x_1, x_2, \ldots$ and x_k in an increasing order. The first age interval ranges from age 0 to x_1; the *i*-th age interval covers from age x_{i-1} to x_i; and the $(k+1)$-*th* age interval contains age x_k and over.

The decomposition will be carried out by finding a function E_i of past demographic changes in the *i-th* age interval that satisfies:

$$P_2 - P_1 = \sum_{i=1}^{k+1} E_i. \tag{A.1}$$

Key terms and concepts

The basic idea underlying the discrete version of the present method of decomposition is the change in cohort size ratio, which is described below. Suppose that the number of persons aged y grew between t_1 and t_2. The proportional increase is $N(y, t_2)/N(y, t_1)$, which is the ratio at the same age y of two different cohorts that were born $t_2 - t_1$ years apart. The past history of the two cohorts is followed and their size during the same years of age is compared. The cohort size ratio remains constant, say, from age x_1 to x_2 if the two cohorts experience the same set of age-specific death rates and rates of net-migration. The cohort size ratio changes if the two cohorts follow different schedules of mortality and migration. Therefore, the change from age x_1 to x_2 of the ratio of the two cohorts can be considered the impact of changes in mortality and migration during the age interval upon the cohort size ratio. Note that all cohort size ratios above age x_2 are affected by the changes.

For example, the proportional increase from 1970 to 1980 in the number of persons aged 60 is $N(60, 1980)/N(60, 1970)$. The impact of changes in mortality and migration between ages 30 and 50 on the proportional increase at age 60 is represented by:

$$\frac{N(50, 1970)}{N(50, 1960)} - \frac{N(30, 1950)}{N(30, 1940)}.$$

Formal definitions of the cohort size ratio and its change are as follows: Suppose that there are two cohorts—one aged y at t_2 and the other aged y at t_1. The ratio of these two cohorts at age x is given by:

$$R(x, y) = N(x, t_2 - y + x)/N(x, t_1 - y + x) \qquad \text{if } y \geqslant x$$

or

$$R(x, y) = 0 \qquad \text{if } y < x$$

The definition of the change of the cohort size ratio in the *i-th* age interval, denoted by $D_i(y)$, is more complicated, and the following three different cases need to be considered separately.

(*a*) When $i = 1$:

$$D_1(y) = R(x_1, y) - 1 \qquad \text{if } y \geqslant x$$

or

$$D_1(y) = R(y, y) - 1 \qquad \text{if } x_1 > y.$$

(*b*) When $i = 2, 3, \ldots,$ or k:

$$D_i(y) = R(x_i, y) - R(x_{i-1}, y) \qquad \text{if } y \geqslant x_i$$

$$D_i(y) = R(y, y) - R(x_{i-1}, y) \qquad \text{if } x_i > y \geqslant x_{i-1}$$

or

$$D_i(y) = 0 \qquad \text{if } x_{i-1} > y.$$

(c) When $i = k + 1$:

$$D_{k+1}(y) = R(y, y) - R(x_k, y) \qquad \text{if } y \geqslant x_k$$

or

$$D_{k+1}(y) = 0 \qquad \text{if } x_k > y.$$

DECOMPOSITON

These definitions help to specify an appropriate expression for E_i. Consider the following expression:

$$E_i = (G_1/S_2)[\sum_{y=a}^{b} g_1(y)D_i(y) - \sum_{y=0}^{\infty} s_1(y)D_i(y)]$$

where $g_1(y)$ and $s_1(y)$ are as defined earlier. It will be shown that the expression for E_i satisfies equation (A.1). In the above expression, (G_1/S_2) is a scaling factor, the first term in the bracket represents the effects of demographic changes in the *i-th* age interval on the proportional increase in the number of persons aged a to b, and the second term in the bracket represents the effects of demographic changes in the *i-th* age group on the proportional increase of the total population.

In order to show that the above E_i satisfies (A.1), the following (A.2), (A.3) and (A.4) are needed. The proportional increase in the number of persons aged a to b can be decomposed as:

$$\frac{G_2}{G_1} - 1 = \sum_{y=a}^{b} \frac{N(y, t_1)}{G_1} \left[\frac{N(y, t_2)}{N(y, t_1)} - 1 \right]$$

$$= \sum_{y=a}^{b} g_1(y)\,[R(y,y) - 1]. \qquad \text{(A.2)}$$

Similarly, the proportional increase of the total population can be decomposed as:

$$\frac{S_2}{S_1} - 1 = \sum_{y=0}^{\infty} \frac{N(y, t_1)}{S_1} \left[\frac{N(y, t_2)}{N(y, t_1)} - 1 \right]$$

$$= \sum_{y=0}^{\infty} s_1(y)\,[R(y,y) - 1] \qquad \text{(A.3)}$$

It can also be shown that:

$$R(y,y) - 1 = \sum_{i=1}^{k+1} D_i(y) \qquad \text{(A.4)}$$

in three different cases:

(a) If $y \geqslant x_k$, then

$$R(y,y) - 1 = R(y,y) - R(x_k, y) + R(x_k, y) - R(x_{k-1}, y) + R(x_{k-1}, y)$$

$$\ldots - R(x_1, y) + R(x_1, y) - 1$$

$$= [R(y,y) - R(x_k, y)] + \sum_{i=2}^{k} [R(x_i, y) - R(x_{i-1}, y)] + [R(x_1, y) - 1]$$

$$= \sum_{i=1}^{k+1} D_i(y).$$

(*b*) Similarly, if $i = 2, 3, \ldots,$ or k and $x_i > y \geqslant x_{i-1}$, then

$$R(y,y) - 1 = R(y,y) + 0 - R(x_{i-1}, y) + \sum_{j=2}^{i-1} [R(x_j, y) - R(x_{j-1}, y)] + [R(x_1, y) - 1]$$

$$= \sum_{i=1}^{k+1} D_i(y).$$

(*c*) Lastly, if $y < x_1$,

$$R(y,y) - 1 = D_1(y) = \sum_{i=1}^{k+1} D_i(y)$$

because

$$D_i(y) = 0 \text{ for } i = 2, 3, \ldots, k + 1.$$

It follows from (A.2), (A.3) and (A.4) that

$$P_2 - P_1 = \frac{G_1}{S_2}\left[\left(\frac{G_2}{G_1} - 1\right) - \left(\frac{S_2}{S_1} - 1\right)\right]$$

$$= \frac{G_1}{S_2}\left[\sum_{y=a}^{b} g_1(y) \sum_{i=1}^{k+1} D_i(y) - \sum_{y=0}^{\infty} s_1(y) \sum_{i=1}^{k+1} D_i(y)\right]$$

$$= \sum_{i=1}^{k+1} \frac{G_1}{S_2}\left[\sum_{y=a}^{b} g_1(y)D_i(y) - \sum_{y=0}^{\infty} s_1(y)D_i(y)\right].$$

ANNEX II

Alternative expression for the population growth rate

One of the most fundamental accounting identities in demography is

$$r_T(t) = b(t) - d(t) + g(t), \tag{B.1}$$

where $r_T(t)$, $b(t)$, $d(t)$ and $g(t)$ are the growth rate of total population, the crude birth rate (CBR), the crude death rate (CDR), and the crude rate of net migration, respectively, at time t.

An alternative expression for the growth rate of total population is given by

$$r_T(t) = \int_0^{\infty} c(x,t)r(x,t)dx \tag{B.2}$$

where $c(x,t)$ is the density function of age x and time t that represents the proportional age distribution of population and $r(x,t)$ is the growth rate of population at age x and time t.

As shown above, the age-specific growth rate is determined by past demographic changes as follows:

$$r(x,t) = r_B(t-x) - \int_0^x \frac{\partial \mu(a,u)}{\partial u}da + \int_0^x \frac{\partial m(a,u)}{\partial u}da, \tag{B.3}$$

where $r_B(t)$ is the growth rate of the number of birth at time t and $\mu(a,u)$ and $m(a,u)$ are the death rate and rate of net migration, respectively, at age a and time u. The derivatives in the above equation are assessed at $u = t - x + a$.

The growth rate of the number of births can be decomposed as

$$r_B(t) = r_T(t) + r_b(t) + r_f(t), \tag{B.4}$$

where $r_b(t)$ is the growth rate of the ratio of CBR to the total fertility rate (TFR) at time t and $r_f(t)$ is the growth rate of the total fertility rate at time t. The CBR/TFR ratio represents effects on the crude birth rate of relationships between the population age structure and the age pattern of fertility (Horiuchi, 1991).

By substituting equations (B.3) and (B.4) into equation (B.2), the growth rate of total population is expressed as

$$\begin{aligned} r_T(t) = & \int_{-\infty}^{t} c(t-u,t)r_T(u)du + \int_{-\infty}^{t} c(t-u,t)r_b(u)du + \int_{-\infty}^{t} c(t-u,t)r_f(u)du \\ & - \int_0^{\infty}\int_{-\infty}^{t} c(a+t-u,t)\frac{\partial \mu(a,u)}{\partial u}duda \\ & + \int_0^{\infty}\int_{-\infty}^{t} c(a+t-u,t)\frac{\partial m(a,u)}{\partial u}duda. \end{aligned} \tag{B.5}$$

The two expressions represent two different viewpoints. Equation (B.1) describes the population growth in terms of the *current* population dynamics: the size of a population grows when the number of births and in-migrants exceeds the number of deaths and out-migrants. Equation (B.5), on the other hand, describes the *current* population growth as a result of *past* changes in population size, structure, fertility, mortality and migration. It can be held, for example, that the total size of population grows if the number of children, the number of working-age adults and the number of the elderly increase together; and the growth of a particular age group may result from an increase in the number of births during the period when the cohort currently in those ages were born, a mortality decline that the cohort had experienced, a past influx of migrants into the cohort, or a combination of these factors.

In practice, it is difficult to fill out all terms of (B.5) with data, because detailed information is needed on the history of the study population for the past, say, 100 years. Nevertheless, the retrospective perspective in equation (B.2) seems to lead to a better understanding of the impact of the momentum of past demographic histories on current population growth.