Population Studies, **49** (1995), 147–163
Printed in Great Britain

The Cohort Approach to Population Growth: A Retrospective Decomposition of Growth Rates for Sweden

SHIRO HORIUCHI*

INTRODUCTION

Demographic conditions at a particular moment in time affect population growth not only during the same period, but also in later years. A well-known example is Keyfitz's population momentum.[1] When a population is growing rapidly, a sudden fall in fertility to replacement level does not result in an immediate cessation of growth. The 'momentum' of past demographic dynamics results in growth continuing. Projections for actual high-fertility populations usually show that the increase due to momentum would be substantial.

In an analysis of current population growth it is important to take past population history into consideration. In India, for example, the crude birth rate fell significantly during the early 1970s, but remained almost unchanged from 1977 to 1985. The major cause of this stagnation was a slowing down of the decline in fertility, but another important factor was the increasing proportion of women at peak childbearing ages.[2] This change in structure was the result of the ageing of those born during the 1950s and 1960s, cohorts in which high fertility and improved child mortality had caused a rapid increase in the number of young children.

In the industrialized world there are more than 20 countries in Europe and East Asia in which the intrinsic growth rate of the population is negative, but the actual growth rate is positive, largely because of an increase in the number of births and reduction of mortality in the past. Cohorts, whose members are currently in middle and old age groups were born at a time when the number of births was increasing rapidly, and they have also benefited from the reduction in mortality throughout their lives. The population of the elderly and of those in middle age is, therefore, still increasing and more than compensates for the decrease in the young population.

These explanations of current population growth in terms of conditions in the past, though plausible, have not been confirmed quantitatively, because methods for measuring the effects of demographic changes on later population growth were lacking. In this paper, we shall develop an equation which will decompose population growth in terms of the effects of past changes, and apply it to the demographic history of Sweden, which is one of the countries in which current population growth rates are positive, whilst intrinsic rates of growth are negative.

* Rockefeller University, 1230, York Avenue, New York. N.Y. 10021. This research was supported by U.S. National Science Foundation Grants BSR87-05047 and BSR92-07293 to Rockefeller University (Principal Investigator: Joel E. Cohen). I am very grateful to Joel Cohen and Lorenzo Moreno for comments on earlier versions of this paper and to Ansley J. Coale for advice on Swedish historical data.

[1] N. Keyfitz, 'On the momentum of population growth'. *Demography* **8** (1971), pp. 71–80.

[2] M. D. Chaudhury, 'Fertility behaviour in India, 1961–86: The stalled decline in the crude birth rate'. In S. N. Singh, M. K. Premi, P. S. Bhatia and A. Bose (eds.), *Population Transition in India* (Delhi, BR Publishing 1989), pp. 89–104.

METHOD

A fundamental accounting identity in demography is the expression for population change:

$$\Delta N = B - D + I - E,$$

where ΔN is the increment in population size during a given period and B, D, I and E are the numbers of births, deaths, immigrants, and emigrants respectively during the same period. It is based on the view that a population is a group of individuals with two methods of entry (births and immigration) and two exits (deaths and emigration); thus any change in the population size is determined by the number of movements through these channels. We may call this the *entrance-exit view of population growth.* Similarly, the growth rate of total population r_T is expressed as:

$$r_T = b - d + i - e,$$

where b, d, i, and e are, respectively, the crude birth rate, the crude death rate, the crude immigration, and the crude emigration rates.

Alternatively, population change may be considered as the sum of the increments in the numbers in all age (or age-sex) groups of the population. The size of an age group changes because the cohort of a given age at the beginning of a period is replaced by a later cohort whose members reach the same age at the end of the period. We call this the *cohort replacement* view of population growth.

The difference between the sizes of cohorts that are compared at the same age is the result of their different demographic history: initial cohort size (original number of births in the cohort), and the age schedules of mortality and migration that its members have experienced between birth and reaching the age when they are being compared. The number of births, in turn, is determined by the size of the population, its age structure, and the age structure of fertility operating in the period when members of the cohort were born.

Therefore, by summing the differences in cohort-size over all ages, the change in the total population may be expressed in terms of its past history, instead of by the current balance of births, deaths, and migration. We shall show that the growth rate of the total population may be regarded as the weighted sum of past changes in five factors: population size, age distribution, fertility, mortality, and migration.[3] This equation will be called the *retrospective decomposition of population growth rate*, and the expression of relative population change shown earlier will be called the *synchronous decomposition of population growth rate.*[4] The difference between the two is rooted in the fundamental

[3] The methodological discussion in this section constitutes an extension of some recent studies on changes in age structure and age-specific growth rates. See S. Horiuchi and S. H. Preston, 'Age specific growth rates: the legacy of past population dynamics'. *Demography* **25** (1988), pp. 429–441; Y. C. Yu and S. Horiuchi, 'Global trends and prospects of aging population structures', in *Economic and Social Implications of Population Aging* (New York, United Nations, 1988); S. H. Preston, C. Himes and M. Eggers, 'Demographic conditions responsible for population aging'. *Demography* **26** (1989), pp. 691–704; G. Caselli and J. Vallin, 'Mortality and population ageing'. *European Journal of Population* **6** (1990), pp. 1–25; S. Takahashi, 'Effects of fertility and mortality changes on aspects of aging in Japan'. (In Japanese, with English abstract), *The Journal of Population Problems* **46** (1990), pp. 1–15; S. Horiuchi, 'Measurements and analysis of cohort size variations'. *Population Bulletin of the United Nations* **30** (1991), pp. 106–124; 'Assessing effects of mortality reduction on population aging' *Population Bulletin of the United Nations* **31** (1991), pp. 38–51; 'Global trends of age distribution, 1950–1990' in *Changing Population Structures 1990–2015* (Geneva, U.N. Economic Commission for Europe, 1992, pp. 4–22).

[4] A special version of the synchronous decomposition, in which the growth rate is separated into the effect of the age distribution and the effect of the net reproduction index, was developed by S. H. Preston, 'Empirical analysis of the contribution of age composition on population growth', *Demography* **7** (1970), pp. 417–432.

dichotomy in demographic method: *period* compared with *cohort* approach.[5] The retrospective decomposition provides a different picture of population growth from that provided by the conventional period approach.

Assume that the population aged x at time t (in years) is a continuous and differentiable function of x and t, $N(x, t)$; where $0 \leqslant x \leqslant \omega$; $-\infty < t < \infty$, and ω is the maximum age at survival. For the sake of simplicity, we consider a population of a single sex. Then, the growth rate of the total population at time t will be:

$$r_T(t) = \int_0^\omega c(x, t)\, r(x, t)\, dx, \tag{1}$$

where $c(x, t)$ is the proportionate age distribution of the population, aged x at time t, and $r(x, t)$ is the growth rate of the population aged x at time t.

The population aged x at time t is given by the product of four terms:

$$N(x, t) = N_T(t-x)\left[\int_\alpha^\beta c(y, t-x) f(y, t-x)\, dy\right] \exp\left[-\int_0^x \mu(y, t-x+y)\, dy\right] \times \exp\left[\int_0^x g(y, t-x+y)\, dy\right], \tag{2}$$

where $N_T(t)$ is the total population at time t, α and β are the lowest and highest ages of childbearing, and $f(x, t)$, $\mu(x, t)$ and $g(x, t)$ are the rates of fertility, mortality, and net migration respectively, at age x and time t. The second and third factors on the right hand side of (2) are the crude birth rate at time $t-x$, and the survival rate of the cohort born at time $t-x$ from birth to age x respectively.

Taking the natural logarithm of Equation (2) and differentiating with respect to time gives an expression for $r(x, t)$, and substituting the result into Equation (1) yields

$$r_T(t) = E_P + E_C + E_F + E_M + E_G, \tag{3}$$

where the Es are the effects of past changes in population size, age distribution, fertility, mortality, and migration respectively. They are given by:

$$E_P = \int_{t-\omega}^t c(t-u, t)\, r_T(u)\, du, \tag{4}$$

$$E_C = \int_{t-\omega}^t \frac{c(t-u, t)}{b(u)} \int_\alpha^\beta f(y, u) \frac{\partial c(y, u)}{\partial u}\, dy\, du, \tag{5}$$

$$E_F = \int_{t-\omega}^t \frac{c(t-u, t)}{b(u)} \int_\alpha^\beta c(y, u) \frac{\partial f(y, u)}{\partial u}\, dy\, du, \tag{6}$$

$$E_M = -\int_0^\omega \int_{t-\omega}^t c(y+t-u, t) \frac{\partial \mu(y, u)}{\partial u}\, du\, dy, \tag{7}$$

$$E_G = \int_0^\omega \int_{t-\omega}^t c(y+t-u, t) \frac{\partial g(y, u)}{\partial u}\, du\, dy, \tag{8}$$

where $b(u)$ is the crude birth rate at time u. The order of integration in Equations (7) and

[5] Note, that both the synchronous and the retrospective equations decompose the overall growth rate into its different components. A synchronous decomposition analysis of *change* in the population growth rate will be found in S. Horiuchi, 'Stagnation in the decline of the world population growth rate during the 1980s'. *Science* **257** (1992), pp. 761–765.

(8) is different from that in Equations (5) and (6) for simplicity of expression. In a stable population E_C, E_F, E_M, and E_G will be zero, and $r_T(u)$ in Equation (4) is equal to the intrinsic growth rate.[6]

Although these equations were given for a single-sex population, they can be changed to a two-sex version by specifying $f(y,u)$ to be the maternal fertility function, and changing $c(y,u)$ (but not $c(t-u,t)$) in Equations (5) and (6) to $p(y,u)$, the function which gives the proportion of the total (two-sex) population at time u, who are women aged y.

It is important to note a fundamental difference between the two approaches to population growth: in the synchronous decomposition, all inward and outward flows contribute positively or negatively to growth; in the retrospective decomposition, it is not the flows themselves, but the *changes* in these flows that increase or decrease the size of the population. The difference could lead to opposite interpretations of the same phenomenon. For instance, an excess of emigration over immigration contributes negatively to population growth in the synchronous decomposition; however, a reduction in the emigration rate contributes positively to population growth in the retrospective decomposition, even when emigration exceeds immigration.

This retrospective decomposition is closely related to population momentum. It can be shown that if age schedules of fertility, mortality, and net migration remain unchanged after time u, we have:

$$r_T(t)-r_S(t) = -\frac{\partial \log \Omega(t)}{\partial t} \quad \text{for } t > u,$$

where

$$\Omega(t) = \frac{S(t)}{N_T(t)},$$

$r_s(t)$ is the intrinsic rate of growth at time t, and $S(t)$ is the stable equivalent population size at time t.[7] Schoen and Kim called $\Omega(t)$ the 'generalized population momentum' which includes Keyfitz's population momentum as a special case.[8] The population momentum, therefore, represents the *consequences* of the difference between actual and intrinsic growth rates for the ultimate size of the stationary population (Keyfitz), or the trajectory towards stable increase (Schoen and Kim). The retrospective decomposition, on the other hand, indicates the effect of past demographic changes on the actual growth rate, and thus clarifies the *causes* of the difference between the two growth rates. Thus, the retrospective decomposition and the population momentum may be considered as two aspects (retrospective and prospective) of the same phenomenon.

RESULTS

Before examining the results of the decomposition, we shall provide an overview of 210 years of Swedish demographic history as a background (Figure 1), using the data described in the Appendix. In Figures 1*a* and 1*b*, we show trends in seven major indicators of population dynamics: the population growth rate, the crude birth rate, the

[6] Relationships between age-specific growth rates and convergence to stability have been discussed by R. Schoen and Y. J. Kim, 'Convergence toward stability is a fundamental principle of population dynamics'. *Demography* **28** (1991), pp. 455–466; 'Covariances, roots and the dynamics of age-specific growth'. *Population Index* **58** (1992), pp. 4–17.

[7] For the stable equivalent population, see N. Keyfitz, *Applied Mathematical Demography* (New York, John Wiley & Sons), 1985. chap. 7.

[8] Schoen and Kim (1991), *loc. cit.* in fn. 6; Keyfitz, *loc. cit.* in fn. 1.

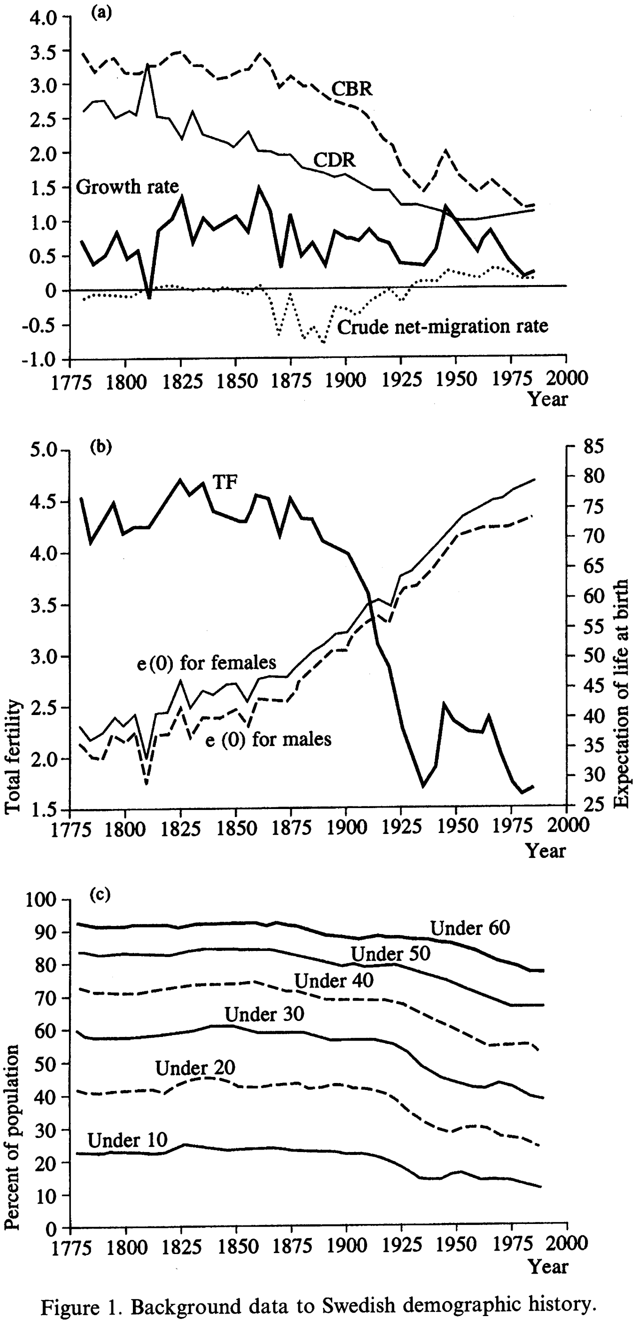

Figure 1. Background data to Swedish demographic history.

crude death rate, the crude net migration rate, total fertility, and life expectancies at birth for females and males, from 1778 to 1987. These data are quinquennial averages. Total fertility fluctuated between 4.2 and 4.7 during the first 100 years, began to decline during the last quarter of the nineteenth century, and reached a trough of 1.7 during the Great Depression of 1933–37. It then rose to above 2.0 during the baby boom of the mid-1940s, only to fall steeply again thereafter, although it has recently risen to a value of 2.12 in 1991. Life expectancy was less than 40 years during the late eighteenth century and slowly increased (with fluctuations) during the early nineteenth century. The increase accelerated towards the end of this century and became less erratic, to slow down after 1950. Crude birth and death rates reflected these trends in total fertility and life expectancy respectively, except for the rise during recent decades in the crude death rate which was due to population ageing. Crude net migration was negative until about 1930, when the direction of the net flow was reversed.

The annual rate of population growth fluctuated within the range from 0 to 1.5 per cent. It reached a peak of 1.46 per cent in 1858–62, because of the fall in the crude death rate, but declined during the late nineteenth century as a result of emigration to the United States. After the end of the emigration boom, it increased again to 0.83 per cent for 1893–97 and then fell to a trough of 0.30 per cent in 1933–37, because of the decline in the crude birth rate. The baby room raised the growth rate to 1.15 per cent for 1943–47, after which the declining trend was resumed. In general, the Swedish growth rate during the nineteenth century was lower than that of many other countries which reached comparable stages of the demographic transition later. This is due to the relatively low initial level of fertility (total fertility less than five children), a slow decline of mortality, and substantial emigration. Although the rate of natural increase fell from the 1880s to the 1930s, it was partly compensated by an increase in the crude migration rate, and, therefore, did not clearly reflect the trend in the growth rate.

Changes in the proportional age distribution are shown in Figure 1*c*. The dominant trend was towards an ageing of the population. The increase in the proportion of the elderly appears to have begun earlier than the decrease in the proportion of young children, probably because the fall in mortality preceded that in fertility. The low fertility of the 1930s and its rise during the 1940s are clearly reflected in subsequent changes of the age structure.

The overall picture of the retrospective decomposition between 1878 and 1987 is shown in Figure 2. Note that the period covered in Figures 2–5 is 100 years shorter than

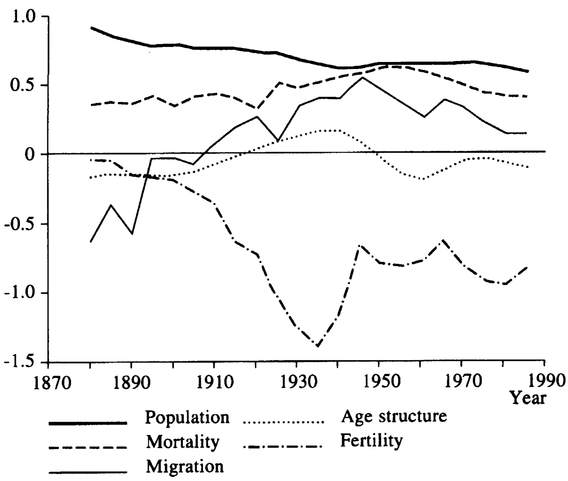

Figure 2. Retrospective decomposition between 1878 and 1987.

in Figure 1. The unit of each effect is the same as that of the annual growth rate. Figure 2 shows, as expected that population growth and falling mortality exerted an upward pressure on later growth rates, whereas the fertility decline exerted downward pressure. The effect of age structure tended to be small and fluctuated around zero. The effect of

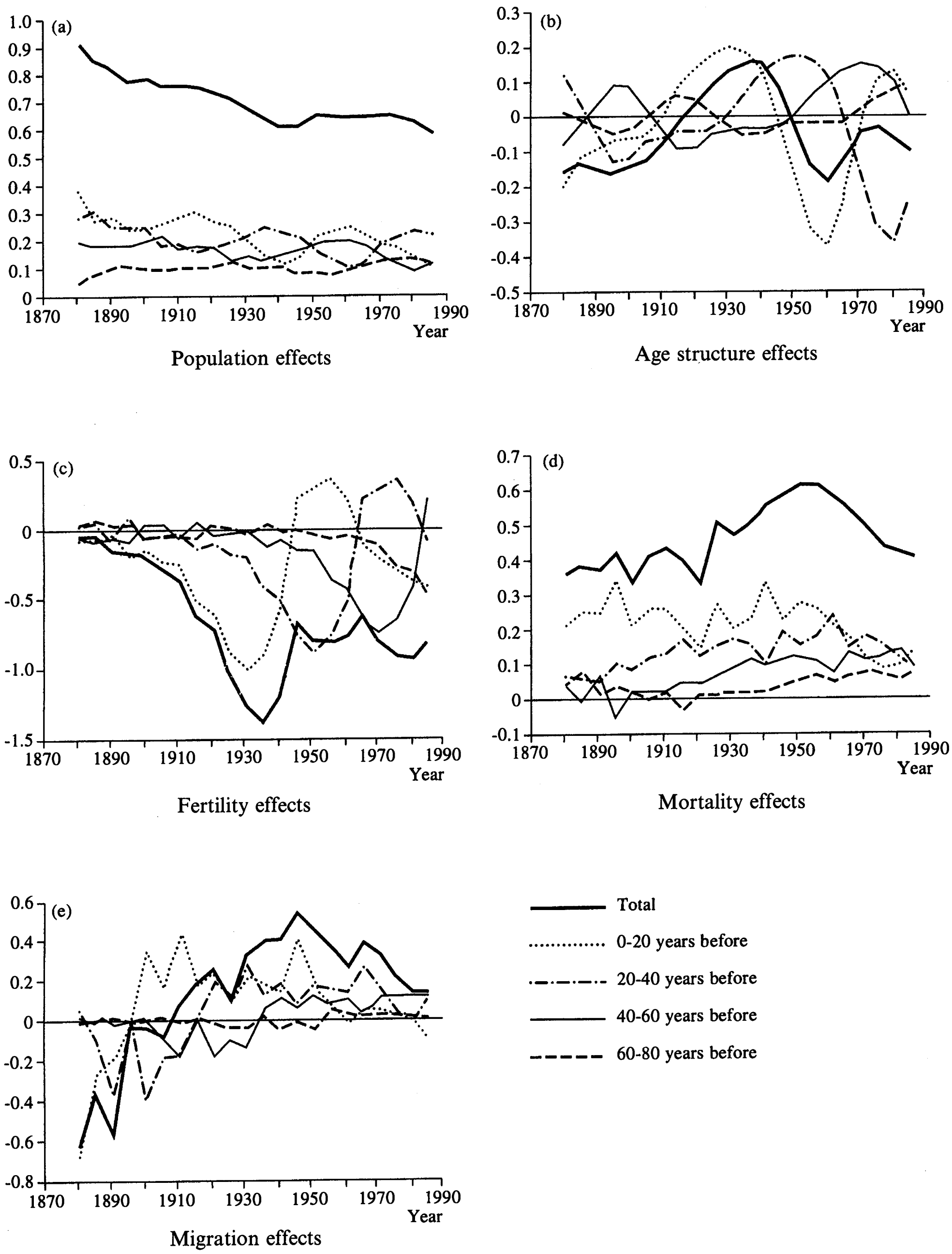

Figure 3. Duration-specific effects of demographic variables between 1878 and 1987.

migration changed sign from negative to positive, to reflect the increase in the net flow since 1880. The strongest positive impact on the growth rate was 0.91, the population effect for 1778–82, the largest negative factor was −1.40, the fertility effect for 1933–37. The largest variations over time were found in the fertility effect, followed by the migration effect.

The impacts of these demographic variables are shown in Figure 3 by the length of time elapsed between the date when the changes occurred and the population growth that they affect. We shall call them duration-specific effects,[9] and group them into four categories: 0–20 years, 20–40 years, 40–60 years and 60–80 years.[10] The effect of changes which had occurred 80 or more years previously is negligibly small and not shown here.[11]

Figure 3a indicates that the total effect of past population growth decreased gradually from 0.91 in 1878–82 to 0.59 for 1983–87, which reflects the long-run trend of the decline in the growth rate. The trajectory is smoother than the erratic curve of the growth rate itself in Figure 1a because the effect is a weighted average of growth rates during the last 100 years. The effect of population changes during the previous 20 years follows a wavy path, which reflects some of the important events in Swedish demographic history: the steep increase in net migration (though still negative) around the turn of the century, the decline in the birth rate during the early twentieth century, the baby boom, and the subsequent resumption of the declining trend by the birth rate. The curve is shifted by 20 years to show the effect of the population increase 20–40 years previously and by 40 years for that of 40–60 years previously.

The total effect of changes in the age structure rose to a peak of 0.16 for 1933–42, and then decreased steeply during the 1940s and 1950s. (Figure 3b). A major determinant of the trend in the duration–specific age structure seems to have been the trend in fertility, a decline leads to a decrease in the proportion of young children in the population and, on balance, increases the proportion at reproductive ages; a rise in fertility has the opposite effect. The trend in the effect of age-structure changes during the preceding 20 years, therefore, appears to be a reverse of the curve of the corresponding fertility effect in Figure 3c, with a slight delay and reduced size.

The total effect of fertility was negative throughout the entire period, because of fertility decline. Figure 3 indicates the downward trend of the effect reflecting the fertility decline during the early twentieth century, as well as its local peak (−0.67 for 1943–47) due to the baby boom. Until about 1930, the trajectory of the total effect of fertility was determined mainly by that of fertility changes during the previous 20 years, because the small variations in fertility during the nineteenth century kept the other duration-specific effects small. Thereafter, the two curves diverged. The fertility effect at durations of 20 years or less became positive because of the baby boom, but the total effect remained negative, because the cumulation of previous fertility declines more than compensated for the positive impact of the baby boom.

The total effect of mortality gradually increased, with fluctuations, to a peak value of

[9] Expressions for duration-specific effects are obtained directly from Equations (4) to (8) by changing the range of integration with respect to time u. The lower and upper limits of the integration are changed from $t-\omega$ and t to $t-v_2$ and $t-v_1$ ($v_1 < v_2$) respectively, for the effect on growth rate at t of changes that occurred between v_1 and v_2 time units (e.g. years) previously.

[10] In this section, the length of time elapsed between demographic changes and the later population growth they affected is approximate. It is shown as the range between two values: the difference between the centre of the first quinquennium of demographic change, and the centre of the period for which population growth has been calculated, on one hand, and the difference between the centre of the last quinquennium and the centre of the growth rate period on the other. For example, as regards the growth rate in 1983–87, changes which date from 1963–67 to 1983–87 are labelled '0–20 years before', and those from 1943–47 to 1963–67 are labelled '20–40 years before'. Although 1963–67 belongs to both periods, no demographic changes have been counted twice. The apparent overlap between the periods is due to the fact that the unit of analysis is not the value of a demographic variable in a quinquennium, but its change between one quinquennium and the next. Similarly, the age ranges of the mortality effects are approximate. Survival from 15–19 to 35–39, for example is labelled as '17.5–37.5' not '15–39', in order to avoid apparently overlapping categories.

[11] The effects of changes which occurred 60–80 years previously shown in Figures 3a–c are relatively flat. Comparison with those of more recent effects shows that the main reason for the flatness is not the relatively small proportion of the population aged between 60 and 80, but the absence of substantial changes in the 60–80 years previously. The sharpest changes in fertility, age structure, and growth rate occurred during the Great Depression and baby boom. Those periods were less than 60 years before 1987, the end of the period of analysis.

0.62 in 1948–52 (Figure 3d). This rise did not result from an accelerated decline in mortality, since the effect of changes in mortality during the previous 20 years remained relatively constant, fluctuating around 0.25 until 1948–52. This rise can be attributed to a cumulation of mortality reductions. Parallel upward slopes, followed by plateaux are seen for the effects with 20–40, 40–60 and 60–80 years of duration. The timings of these curves suggest a start of the substantial decline in mortality during the last quarter of the nineteenth century, and its continuation thereafter, as the period data confirm. Thus, by the mid-twentieth century, several decades of mortality improvement had accumulated. After the peak of 1948–52, however, the total mortality effect, although still positive, follows a downward course. Unlike its rise to a peak value, this decline seems to have been a reflection of the recent trend in mortality, because the total effect paralleled the effect of mortality changes during the preceding 20 years. These patterns in Figure 3d are consistent with trends in Figure 1b which show an acceleration of the increase in life expectancy during the later nineteenth century and a deceleration after 1950.

The mortality effect can be broken down by age. Figure 4 indicates that the impact on population growth of improvements in the mortality of younger children (0–7.5 years) has been greater than that of any other age group. The recent steep increase in the contribution made by the reduction of mortality at old ages (62.5+) is also apparent. The age-duration specific effects of changes in mortality during the last 20 years suggest an increase in the ages at which mortality reduction is important: the peak effect of the mortality of younger children occurred in 1893–97, that of people of younger working ages (17.5–37.5) and older working ages (37.5–62.5) amounted to 0.14 in 1938–43 and 0.08 in 1953–57 respectively, and the effect of old age mortality rose steadily from −0.004 in 1933–37 to 0.10 in 1983–87. It has been pointed out that there was a shift from infectious to degenerative diseases as foci of mortality improvement.[12] The 'ageing' of the mortality effect seems to be consistent with the changing pattern of the epidemiological transition.

The total effect of changes in migration changed sign from negative to positive around 1910 (Figure 3e). This occurred a few decades before the crude net rate of migration turned positive. The difference in timing is due to the fact that net migration began to increase at a time when emigration substantially exceeded immigration. The effect of changes in migration during the preceding 20 years peaked at 0.44 in 1908–12, but the total migration effect during that period was only slightly positive, because it continued to include the emigration boom of the late nineteenth century. The total effect then reached a peak in 1943–47; by then more than 60 years of migration increase had accumulated.[13]

Two tendencies appear to prevail in Figure 3. First the trend of total effect is smoother than that of duration-specific effects, because it is a weighted average of effects of changes that have occurred during the past 100 years. Secondly, although the trend of total effect immediately reflects sharp rises and falls in the demographic variable, a large positive or negative effect generally results from a long-term cumulation of changes in the same direction.

The total population growth rate is the sum of age-specific growth rates, weighted by the age distribution. The trend in each of the five effects can thus be decomposed further into changes in its direct impact on age-specific growth rates and changes in the age distribution. We call the latter the 'weighting factor' effect to avoid confusion with the

[12] S. J. Olshansky and A. B. Ault, 'The fourth stage of the epidemiologic transition: the age of delayed degenerative diseases'. *Milbank Memorial Fund Quarterly* **64** (1986), pp. 355–391; R. G. Rogers and R. Hackenberg, 'Extending epidemiologic transition theory. A new stage'. *Social Biology* **34**, (1987), pp. 234–243.

[13] Trends of age-specific migration effects were similar and are not shown here.

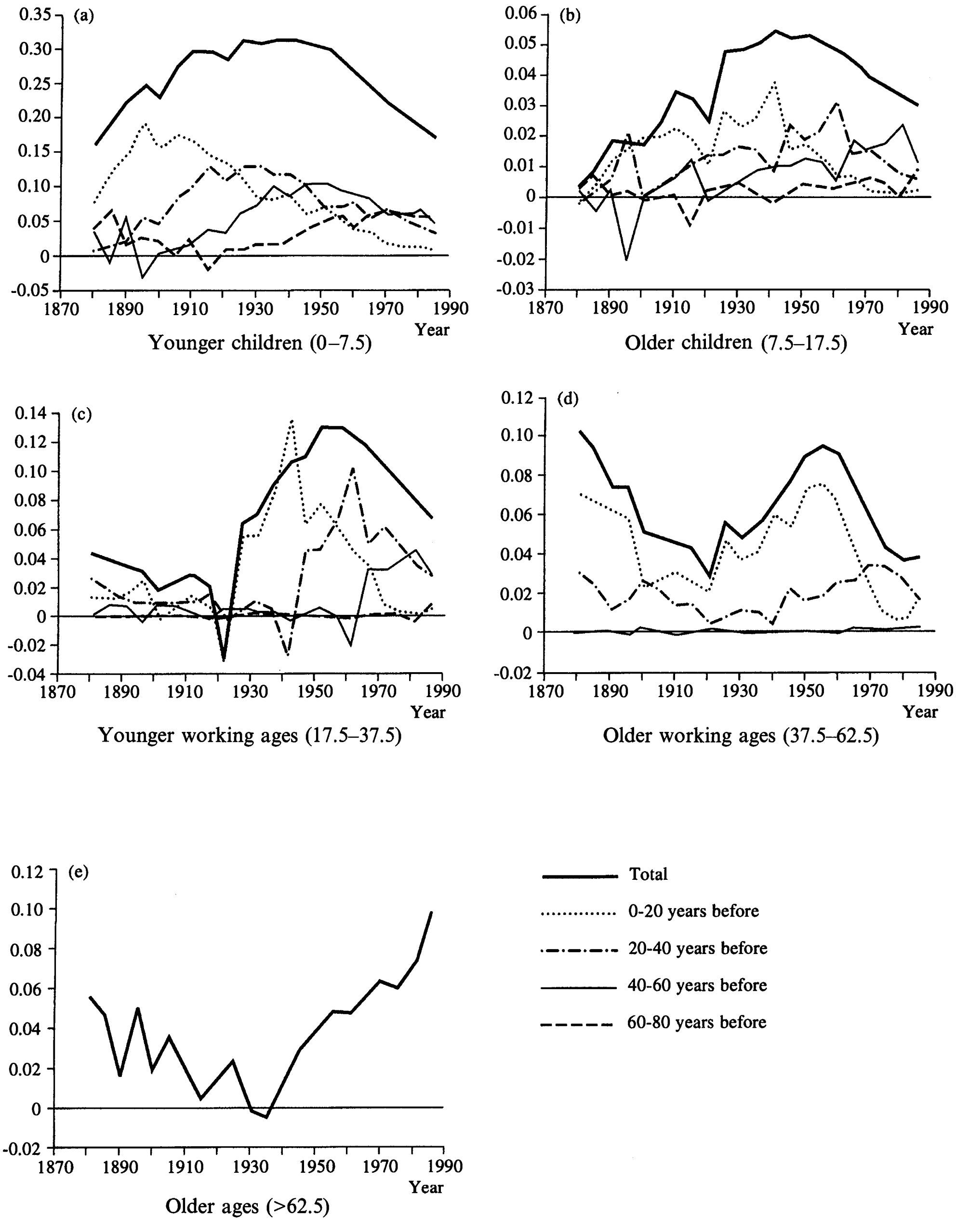

Figure 4. Mortality effects by age and duration between 1878 and 1987.

age-structure effect defined as E_C in Equation (5); the former may be called the 'main effect'. In Figure 5 cumulated changes in the five major effects since 1878–82 have been decomposed into the two components.[14] Overall, the effects of the weighting factor tend to be smaller, and their trends smoother, than the corresponding main effects.

[14] Kitagawa's method without interaction terms was used for each pair of consecutive quinquennia, and the results were cumulated. See E. M. Kitagawa, 'Components of a difference between two rates', *Journal of the American Statistical Association* **50** (1955), pp. 1168–1194.

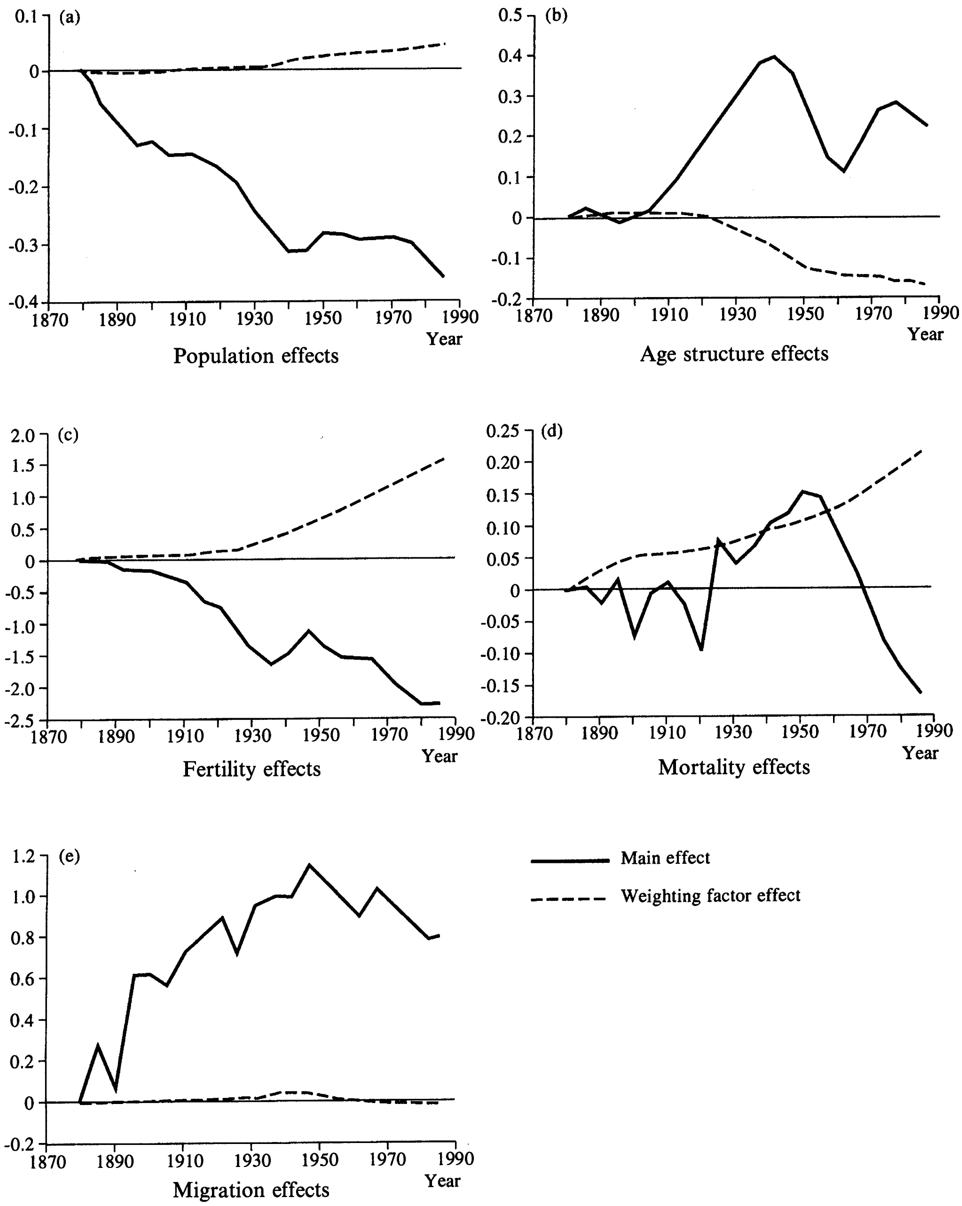

Figure 5. Cumulated changes in the five major effects between 1878–1987.

Population-ageing increased the weighting factor components of the population, fertility, and mortality effects by assigning increasing weights to older age groups. Members of these cohorts were born when growth rates were relatively high and fertility declines moderate. They benefited from both the steep decline in child mortality in the old days, and the substantial reduction in the mortality of old age during recent years. Population ageing exerted a downward pressure on the age structure effect by giving smaller weights to young cohorts. When members of these cohorts were born, the proportion of children was falling, and, on balance, this raised the proportion of the population of reproductive age.

So far, we have followed trends of different demographic effects separately. Another way of looking at the results is to examine these effects together for selected periods. The composition of growth rate for three selected periods, 1883–87, 1933–37, and 1983–87 is shown in Figure 6. Growth rates for the periods were 0.67, 0.30 and 0.21 per cent

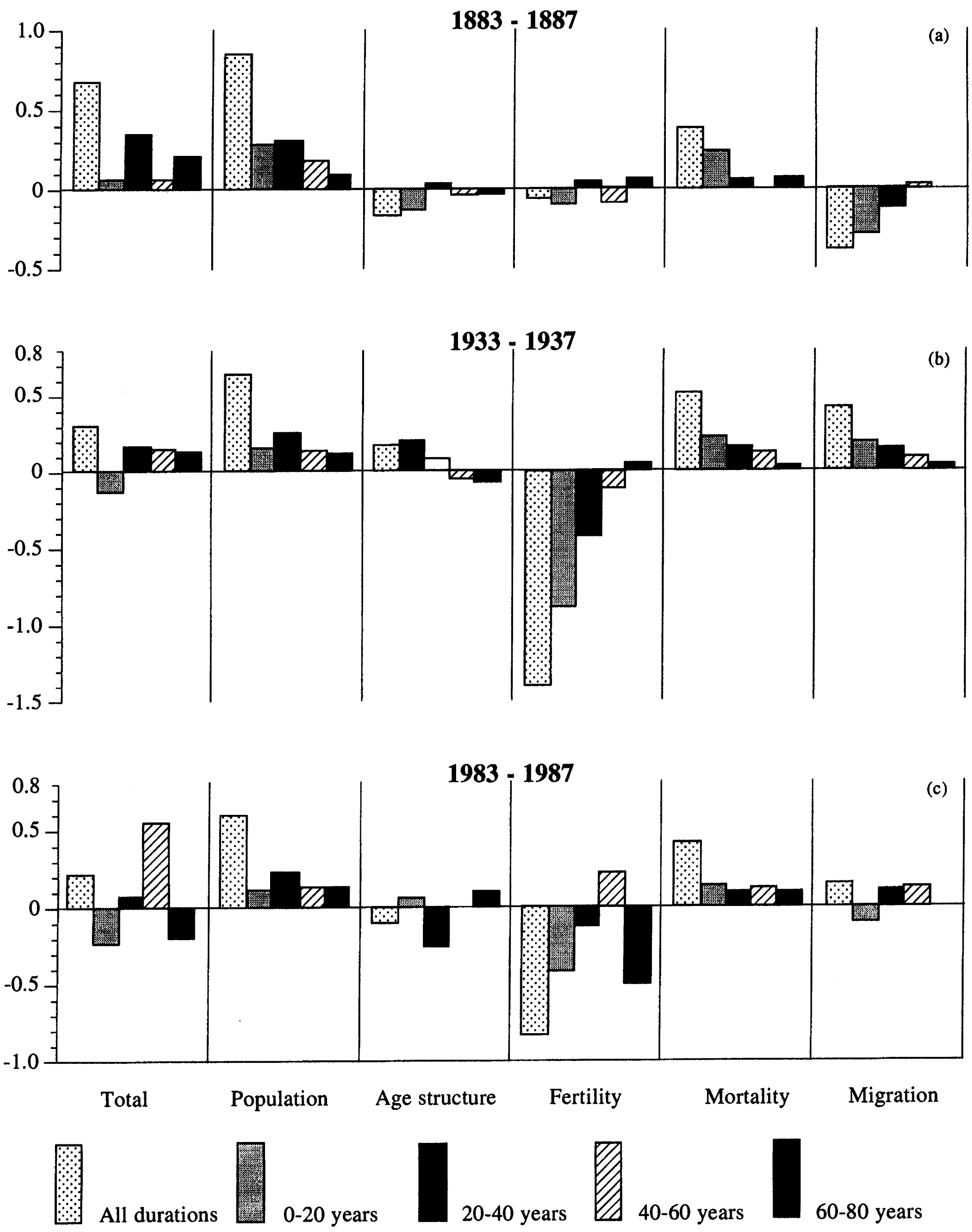

Figure 6. Composition of growth rate for three selected periods: (a) 1883–87; (b) 1933–37; (c) 1988–87.

respectively. The period 1883–87 had a 'quasi-stable' profile; the population effect was strongest (0.85) followed by a moderate effect of mortality reduction (0.38); in contrast the fertility effect was small (−0.05) because of the lack of a consistent trend in previous decades. A deviation from quasi-stability was the substantial migration effect (−0.37).

In particular, migration changes during the previous 20 years, when the crude annual rate of migration fell from −0.18 to −0.58 per cent between 1863–67 and 1883–87, resulted in a reduction of the growth rate in 1883–87 by 0.27.

The period 1933–37 when total fertility reached a trough of 1.7 was marked by a pronounced fertility effect (−1.40), which reflected the decline in fertility during the past 60 years. Effects of more recent fertility reductions tended to be considerably larger, which suggested that the fall had accelerated. Nevertheless, the negative fertility effect was overridden by the sum of the other four effects, all of which contributed positively to the growth rate. Cumulated improvement in child mortality during the last 60 years (0.35), and reduction in adult mortality during the last 20 years (0.12) made the total mortality effect significantly greater than half a century previously. The positive migration effect (0.40) was due to the increase in net migration that began during the late nineteenth century.

The strongest effect in 1983–87 was that of fertility (−0.82), but this was again outweighed by the joint effects of population, mortality, and migration.[15] The negative effect of fertility was caused by recent declines in fertility (−0.42 for the last 20 years), as well as by those during the early twentieth century (−0.49 for 60–80 years ago).

Changes in the profile of the mortality effect merit special attention (Figure 7). The total mortality effect was 0.38 in 1883–87, and its largest duration-specific component was 0.12 for child mortality during the preceding 20 years. Then the total mortality effect increased to 0.50 in 1933–37, mainly as a result of the cumulation of several decades of child-mortality reduction. The profile of the mortality effect changed substantially during the following half-century. In 1983–87, the reduction of child-mortality during recent periods made smaller contributions: 0.01 for the last 20 years, but 0.06 for 60–80 years previously. The largest age-duration-specific effect was 0.10 for mortality in old age during the previous 20 years. The pattern of the contribution of mortality to population growth shifted during the 100 years from 'recent child-mortality reduction' to 'old age mortality decline in the recent past and child-mortality improvement in the remote past'.

Total fertility was below two in 1933–37 and 1983–87. The profiles for these periods provide quantitative explanations for the puzzling combination of positive actual and negative intrinsic growth rates. In both periods, the joint effect of all changes that occurred 0–20 years previously were negative (−0.14 for 1933–37, and −0.23 for 1983–87) as a result of previous fertility declines. These recent negative effects, however, were exceeded by the positive effects of changes which occurred 20 or more years previously, as is shown in the extreme left section of Figure 6. For 1933–37, the remote positive effects came mainly from changes in population size (0.47), child mortality (0.24), and migration (0.19) during the previous 20–60 years. For 1983–87, it was the strong effect (0.55) of all changes which occurred 40–60 years previously, that kept the growth rate positive. The rise in fertility during the baby boom between 1933–37 and 1943–47 was particularly important, it pressed the growth rate for 1983–87 upward by 0.52.

In summary, our analysis has shown that not only events which occurred during the recent past, but also those which occurred several decades previously can exert

[15] The positive effect of population growth 20–40 years previously, the negative effect of age structure 20–40 years previously and the positive effect of changes in fertility 40–60 years previously are mainly due to the baby boom. The three relatively strong effects are split into two different duration-categories of 20–40 and 40–60 years, because the baby boom peak occurred during the mid-1940s, almost exactly 40 years before 1983–87. The rise of fertility to its peak belongs to the '40–60 years previously' category, but the changes in population growth and age structure caused by the baby boom fell mainly into the '20–40 years previously' category.

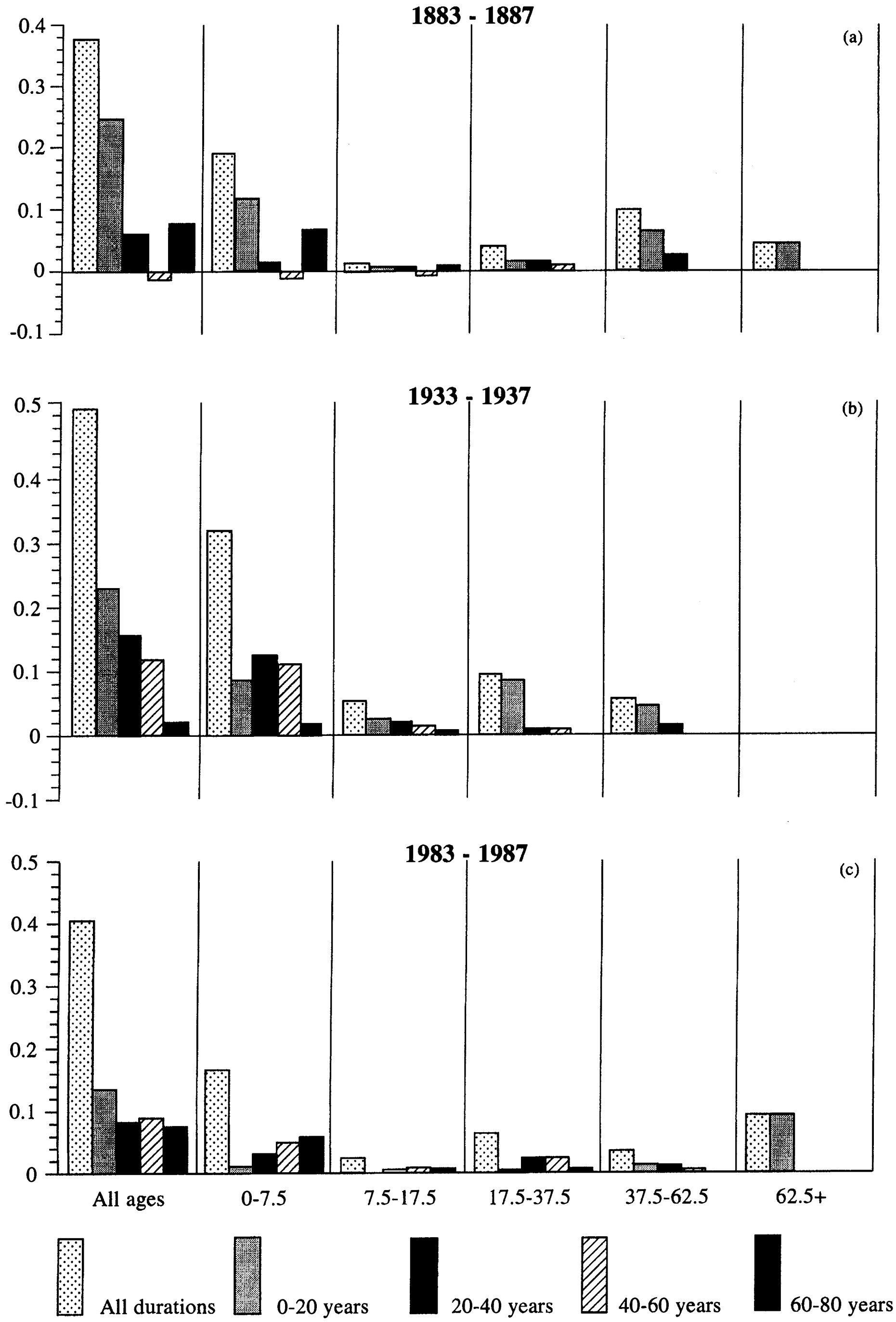

Figure 7. Changes in the mortality effect: (a) 1883–87; (b) 1933–37; (c) 1983–87.

substantial impacts on the current growth rate. The decomposition results reflect typical changes in the demographic and epidemiological transitions, such as a fall in fertility, decline in the growth rate, reduction of mortality, and a shift in the improvement of mortality to older ages. They also show traces of some historical events, such as the Great Depression, the baby boom of the mid-1940s, emigration to the United States during the late nineteenth century, and immigration during the middle and late twentieth century. The results indicate the value of the retrospective decomposition method for measuring and analyzing the effect of demographic changes on later population growth.

DISCUSSION

The use of the method presented is not necessarily limited to the few countries for which there exist long time-series of available data, and the total growth rate of the population is not the only possible subject for retrospective decomposition. If data are available for a few decades only, the method can easily be adjusted to the data structure. It will decompose the growth rate into the effects of demographic changes that have occurred during the period for which data were available, together with a residual term to represent the combined effects of more remote events in the past. Secondly, the demographic history of a country in which detailed and accurate data are lacking may be reconstructed on the basis of some reasonable assumptions, models, or indirect techniques, and decomposition can then be applied to the reconstructed history. Thirdly, if the interest is not in the thorough decomposition of the growth rate but in the effects of a particular event (e.g. the baby boom) on later population growth, the method can be used as far as data are available. Fourthly, the improvement of demographic data collection and quality in many countries will gradually increase the number of countries to which retrospective decomposition can be applied. Finally, the method presented can easily be modified for the decomposition of absolute increments in the total population, particular age groups, and other sub-populations, as well as for the decomposition of changes in the age structure.[16]

We should caution the reader that the method does not deal with some relationships between demographic changes and later population growth. It does not measure *indirect* effects. Equations (4) to (8) express *direct* effects of demographic changes on cohort size, but not *indirect* effects which work through E_P and E_C; demographic changes affect population growth and age distribution in later years, which, in turn, influence population growth in the future. For example, the baby boom contributes not only to current growth in the middle-aged population (i.e. the baby boomers), but also to that of the young population (i.e. their children). The latter effect is indirect; it works through changes in the age structure produced by the ageing of the baby boomers. Although indirect effects in Sweden have not been measured in this study, they can be calculated straightforwardly by linking together some of the equations given earlier.

Secondly, the method does not deal with the effects of past demographic changes on the growth rate through the *current* age distribution. Changes in the weights affect the growth rate trend, as is shown in Figure 5. Although changes in the age distribution are the result of past demographic changes, the present method does not relate them to each other.[17]

Finally, the effects of demographic changes on later population growth may have broader meanings than is implied in the retrospective decomposition. Suppose that a

[16] For cohort approaches to the decomposition of changes in the age structure, see Caselli and Vallin, *loc. cit.* in fn. 3; Takahashi, *loc. cit.* in fn. 3, and Horiuchi, Assessing effects... (1991) *loc. cit.* in fn. 3.

[17] As is shown in the works cited in fn. 16, changes in age distribution can be decomposed into past population dynamics. It seems impossible, however, to combine the procedure with the present method, because they work at different levels of analysis. The present method is based on the decomposition of the age-specific growth rate. It can be extended to decomposition of changes in the age-specific growth rate, but results of the second-derivative level of analysis would be difficult to interpret. The other procedure decomposes changes in the proportion of the population by age, but not the proportion itself.

decline in child mortality were to reduce the desired number of children, which would, in turn, slow down population growth. Such linkages through behavioural mechanism are beyond the scope of the present paper.

APPENDIX

Data

The application of Equations (3)–(8) to an actual population requires the availability of a long time series of data on population, births, deaths, and migration. The demographic history of the past must be traced back to the date of birth of the oldest person in the population. If the maximum life span is 100 years, then data for the previous 100 years are needed to decompose the growth rate for a given period. To analyze the trend in the growth rate during the last 50 years, 150 years of data would be needed, beginning with the births of individuals who were 100 years old 50 years ago. Sweden, with its long history of detailed and accurate demographic data appears to meet these requirements better than any other country.

Keyfitz and Flieger assembled official Swedish statistics from 1778 to 1962 on population by age and sex, deaths by age and sex, and births by age of the mother and sex of child, and tabulated them for successive quinquennia in quinary age groups.[18] The data are very convenient to use in this study. In addition, data for the period 1963–87 were taken from the annual publications of the Central Bureau of Statistics and arranged in Keyfitz and Flieger's format. The entire data set covers 210 years of Swedish demographic history.

Sundbärg has analyzed Swedish historical statistics and adjusted the data for some early periods (before 1816) to allow for estimated under-reporting.[19] Although his corrections were not incorporated in Keyfitz and Flieger's data, they have been accepted by the Central Bureau of Statistics.[20] In this study, we have adjusted Keyfitz and Flieger's data for Sundbärg's estimates.

The discrete version of the retrospective decomposition requires that the oldest age interval, which is usually open, be regarded as closed. Selection of the highest age for the analysis has a trade-off: choice of a higher age reduces errors due to the survival of persons older than that age, choice of a younger (though still advanced) age lengthens the period of trend analysis. Considering the recent increase in the very old population, the highest age group for this study was taken to be 95–99. Inclusion of centenarians in this age group biases the results of the decomposition only slightly. Since those born at the beginning of 1778 who survived to 1878 were 100 years old at the beginning of 1878, the decomposition of the growth rate begins in 1878.

Although data for very old age groups have been tabulated in recent official statistical publications, the oldest age group shown by Keyfitz and Flieger is '85 and over'. We, therefore, need to estimate the age distribution of the population aged 85 years and over between 1878 and 1962. Cohorts aged 80–84 have been forward-projected to age '100 and over' on the assumption that period mortality schedules at old ages follow Gompertz's law, and that migration in these cohorts is zero. The population aged 85 and over has then been distributed according to the projected age composition.[21]

The conversion of Equations (4)–(8) into discrete form is relatively straightforward. All functions of time and age are calculated for successive quinquennia and for either quinary age groups (f and c functions) or for five-year birth cohorts (μ and g functions). Keyfitz and Flieger have estimated the average population by age and sex for each quinquennium, and the population by age and sex at the beginning and end of this period can be geometrically interpolated.

[18] N. Keyfitz and W. Flieger, *World Population. An Analysis of Vital Data.* (Chicago, University of Chicago Press, 1968).

[19] G. Sundbärg, *Bevölkerungsstatistik Schweden's 1750–1900* (Stockholm, National Central Bureau of Statistics, 1976).

[20] National Central Bureau of Statistics, *Historical Statistics of Sweden* Part I: *Population. Second Edition, 1720–1967* (Stockholm, 1969); E. Hofsten and H. Lundström, *Swedish Population History: Main Trends from 1750 to 1970* (Stockholm, National Central Bureau of Statistics, 1976).

[21] Age-related mortality increases tend to slow down at very old age. See, for example, S. Horiuchi and A. J. Coale, 'Age patterns of mortality for older women: An analysis using the age-specific rate of mortality change with age'. *Mathematical Population Studies* **2** (1990), pp. 245–267. Thus, the procedure we have used may underestimate the proportion in older age groups. However, since the proportion aged 85+ between 1878 and 1962 was small, the effects of such errors on the growth rate of the population as a whole are expected to be insignificant.

The use of discrete data makes an inconsistent treatment of the r_T function unavoidable: the growth rate in Equation (3) is obtained directly from two population totals, which are five years apart whereas the growth rate on the right hand side of Equation (4) needs to be calculated from the average population sizes of two consecutive quinquennia. The decomposition of the change in the crude birth rate in Equations (5) and (6) has been performed by using Kitagawa's method without interaction terms.[22]

Deaths in quinary groups have been split into five-year cohorts on the assumption that age distributions are locally stable, and that the mortality schedule follows Gompertz's law. Net migration by cohort is estimated as the difference between the decrement in the cohort size, and the number of deaths in the cohort.[23] Mortality and net migration are assumed to be independent. Although many terms enter into the right-hand side of Equation (3), the maximum absolute errors in the percentage annual growth rates (the computed difference between both sides of the equation) for the 22 quinquennia is 0.0011, and more than half the errors are 0.0001 or less, which suggests that the computational assumptions and procedures adopted in this analysis are reasonable.

[22] The assumption of linear changes, which underlies the omission of interaction terms, seems to provide a reasonable approximation in this case, where the period of change is only five years, so that each change is reasonably small (see Kitagawa *loc. cit.* in fn. 14).

[23] Overall, the net migration estimates agree well with available official statistics on immigration and emigration. A rigorous comparison is, however, impossible because of the difference in the data formats. Hofsten and Lundröm's (*op. cit.* in fn. 20), for example, tabulated immigrants and emigrants by *five-year age groups* for *ten-year periods* between 1851 and 1970. Estimates in this study, however, are obtained for *five-year birth cohorts* in *five-year periods*. Hofsten and Lundström's ten-year periods start with years ending in the digit '1', but our five-year periods start with years ending in digit '3' or '8'.